Lecture 6

Vortex Induced Vibrations

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Aeroelasticity

= Study of the interaction of **inertial**, **structural** and **aerodynamic** forces

Applications on aircrafts, buildings, surface vehicles etc.

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f_a(V_\infty, \dot{x}(t), x(t)) \]
Flow separation

Airfoil/wing aeroelasticity
• Divergence
• Flutter

Bluff-body aeroelasticity
• Vortex Induced Vibration
• Galloping
Flow visualization of vortices shed behind a cylinder
VIV

... induce vibrations

Vortex-induced vibrations
Vortex Induced Vibrations

Dimensional analysis
Examples of VIV
Flow around static cylinders
Effect of a prescribed motion of the cylinder
Free vibration: VIV
Modelling VIV
Lock-in mechanism
Vortex Induced Vibrations

Dimensional analysis

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Dimensional analysis

Non-dimensional variables

- **Geometry**
  \[
  \frac{l}{D} = \text{length} / \text{width}
  \]

- **Dimensionless amplitude**
  \[
  \frac{A}{D} = \text{vibration amplitude} / \text{diameter}
  \]

- **Reduced velocity**
  \[
  U_r = \frac{U_\infty}{fD}
  \]
  - **Large** \( U_r \rightarrow \)** Quasi-steady assumption
  - **Small** \( U_r (< 10) \rightarrow \)** Strong fluid-structure interaction
Dimensional analysis

- **Mass ratio**  
  \[ m_r = \frac{m_S}{m_F} = \frac{m_S}{\rho \pi D^2 / 4} \]  
  = susceptibility of flow induced vibrations

  \[ m_S = \text{mass (p.u. length) of the structure [kg/m]} \]  
  \[ \rho = \text{fluid density [kg/m}^3] \]

- **Reynolds number**  
  \[ \text{Re} = \frac{U_\infty D}{\nu} \]  
  = inertial / viscous forces

  \[ \nu = \text{kinematic viscosity of the fluid [m}^2/\text{s]} \]

  → Measure of the boundary layer thickness

  → Measure of the transition between laminar and turbulent flows

- **Mach number**  
  \[ Ma = \frac{U_\infty}{c} \]  
  = tendency of the flow to compress when it encounters a structure

  In this course: incompressible flows → Ma < 0.3
Dimensional analysis

- **Damping factor** (classically) 
  \[ \eta_S = \frac{\text{energy dissipated per cycle}}{4\pi \times \text{total energy of the structure}} \]

When Fluid-Structure interactions are concerned

→ “reduced damping” or Scruton number \( \approx m_r \times \eta_S \)
Dimensional analysis

Motion of a linear structure in a subsonic, steady flow

Described by:

- Geometry
- Reduced velocity
- Dimensionless amplitude
- Mass ratio
- Reynolds number
- Damping factor

\[ \frac{A}{D} = F\left(\frac{l}{D}, U_r, \text{Re}, m_r, \eta_S\right) \]
Vortex Induced Vibrations

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Examples of VIV
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Modelling VIV
Lock-in mechanism
VIV examples – in air

- Bridges
  - Great-Belt bridge (Denmark)

- Chimneys
  - Burj Khalifa (Dubai)

- High rise buildings
VIV examples – in air

- Humen bridge (China)
VIV examples – in air

- Great-Belt bridge (Denmark)

Unacceptable VIV motion (~ 40cm) during final construction stage at low (and frequent) wind speeds (~8m/s)
VIV examples – in air

(Frandsen 2001)

<table>
<thead>
<tr>
<th>Peak-event on day: (Fig. 4)</th>
<th>Type</th>
<th>Wind speed $U_{10}$ (m/s)</th>
<th>Natural frequency $n_e$ (Hz)</th>
<th>&quot;Mode&quot; (Fig. 7)</th>
<th>Reduced velocity $U_r$ (–)</th>
<th>Turbulence intensity $I_U$ (%)</th>
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<td>0.205</td>
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<td>1.7–1.8</td>
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<td>4 (○)</td>
<td>3</td>
<td>6.9–7.3</td>
<td>0.205</td>
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<td>0.94</td>
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<td>5 (⊗)</td>
<td>6.1–6.6</td>
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<td>0.6</td>
<td>7.7–8.4</td>
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<td>6 (△)</td>
<td>1,2,3</td>
<td>4.5</td>
<td>0.13</td>
<td>3</td>
<td>1.1</td>
<td>22.2</td>
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(Larsen 2000)
VIV examples – in air

- Chimneys

Belgium  UK
VIV examples – in air

• Spheres
VIV examples – in water / 2 phases flows

- Risers

- Heat exchangers

source: Imperial college

source: Sandvik
From 3D to 2D

- Vortex shedding is essentially a 3D phenomenon
  → Flow components in the Z-direction
  → Geometric variation of the structure
  → Dynamic behaviour of the structure

- 2D analysis as a starting point to understand VIV:
  Even on 3D structures, 2D approaches are carried out (flow visualization, load analysis)

- A/D >> 2D dominated

  Structural motion takes the control of aerodynamics
Dimensional analysis

Motion of a 2D linear structure in a subsonic, steady flow

Described by: \[ \frac{A}{D} = F(U_r, Re, m_r, \eta_s) \]

Other parameters:
• Surface roughness of the structure (for rounded shapes)
• Turbulence intensity of the upstream flow

→ Effect on the wake, hence on the lift force and the VIV

Great Belt bridge
Vortex Induced Vibrations

Dimensional analysis
Examples of VIV
Flow around static cylinders
Effect of a prescribed motion of the cylinder
Free vibration: VIV
Modelling VIV
Lock-in mechanism
Flow around a static cylinders

Lifting cylinder (potential flow theory)

\[ C_p = 1 - \left[ 4 \sin^2 \theta + \frac{2 \Gamma \sin \theta}{\pi RV_\infty} + \left( \frac{\Gamma}{2\pi RV_\infty} \right)^2 \right] \]

\[ C_l = \frac{\Gamma}{RV_\infty} \]

\[ \rightarrow \text{Steady lift force} \]
\[ \rightarrow \text{No drag (D’Alembert Paradox)} \]

Viscous effects \(\rightarrow\) Flow separation
Flow around a static cylinders

Creepig flow: vorticity created in the BL is totally dissipated near the body.

Increasing Re = inertial / viscous forces

Effect of viscosity in the vicinity of the body.
Large amount of viscosity not dissipated near the body.

Flow regimes:

- \( Re < 5 \) \( \): Regime of unseparated flow
- \( 5 \leq Re < 40 \) \( \): A fixed pair of Föppl vortices in wake
- \( 40 \leq Re < 90 \) and \( 90 \leq Re < 150 \) \( \): Two regimes in which vortex street is laminar
- \( 150 \leq Re < 300 \) \( \): Transition range to turbulence in vortex
- \( 300 \leq Re < 3 \times 10^5 \) \( \): Vortex street is fully turbulent
- \( 3 \times 10^5 \leq Re < 3.5 \times 10^6 \) \( \): Laminar boundary layer has undergone turbulent transition and wake is narrower and disorganized
- \( 3.5 \times 10^6 \leq Re \) \( \): Re-establishment of turbulent vortex street

Fig. 3-2 Regimes of fluid flow across smooth circular cylinders (Lienhard, 1966).
Flow around a static cylinders

How vortex shedding appears?

- Flow speed outside wake is much higher than inside
- Vorticity gathers in upper and lower layers
- Induced velocities (due to vortices) causes this perturbation to amplify

→ Origin of the vortex shedding process = shear layer instability
   (purely inviscid mechanism)

Vortex shedding = global instability : the whole wake is affected
   = robust : vorticity is continuously produced
   @ a well defined frequency (see Strouhal)
Flow around a static cylinders

**Strouhal number**

\[ St \equiv \frac{f_{VS}D}{U_{\infty}} \]

- \( f_{VS} \) = shedding frequency
- \( D \) = cross dimension
- \( U_{\infty} \) = free stream velocity

\[ \Rightarrow St \sim \text{cste} \]

in a limited range of Reynolds numbers!

\[ \Rightarrow \text{Linear dependency of the shedding frequency with free stream velocity} \]
Flow around a static cylinders

Aerodynamic forces

→ Impact of the shed vortices on the aerodynamic forces

→ Lift force fluctuations at $f_{VS}$

→ Drag force fluctuations at $\sim 2 f_{VS}$
Flow around a static cylinders

Aerodynamic forces

- Lift coefficient is an image of the fluctuating vorticity in the wake (where the dominant frequency is $f_{VS}$)

  $\rightarrow C_L(t)$ is characterized by $f_L$ and $C_L^{rms}$

  $\rightarrow$ Dimensionless Strouhal number $\quad St = \frac{f_L D}{U_\infty}$

  $$C_L(t) = \frac{L(t)}{1/2 \rho U_\infty^2 D} = \sqrt{2} C_L^{rms} \sin(\omega_L t) = \sqrt{2} C_L^{rms} \sin\left(\frac{St \ 2\pi U_\infty}{D} t\right)$$

- Drag coefficient : typically $f_D \sim 2 \ f_{VS}$

  $$C_D(t) = \frac{D(t)}{1/2 \rho U_\infty^2 D} = \sqrt{2} C_D^{rms} \sin(\omega_D t) = \sqrt{2} C_D^{rms} \sin(2\omega_L t) = \sqrt{2} C_D^{rms} \sin\left(\frac{2St \ 2\pi U_\infty}{D} t\right)$$
Flow around a static cylinders

Aerodynamic forces

Strongly Reynolds dependent!

\[ C_L(t) = \frac{L(t)}{1/2 \rho U_\infty^2 D} = \sqrt{2} C_{L}^{rms} \sin(\omega_L t) \]
\[ = \sqrt{2} C_{L}^{rms} \sin\left(\frac{St \cdot 2\pi U_\infty}{D} t\right) \]

\[ C_D(t) = \frac{D(t)}{1/2 \rho U_\infty^2 D} = \sqrt{2} C_{D}^{rms} \sin(\omega_D t) = \sqrt{2} C_{D}^{rms} \sin(2\omega_L t) \]
\[ = \sqrt{2} C_{D}^{rms} \sin\left(\frac{St \cdot 2\pi U_\infty}{D} t\right) \]
Vortex Induced Vibrations

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Modelling VIV
Lock-in mechanism
Oscillating cylinder

→ Cylinder forced to move

Imposed motion : \[ y(t) = A \sin(2\pi f_f t) \]

- \( A = \text{Amplitude} \)
- \( f_f = \text{Forcing frequency} \)

Order of magnitude of the diameter (D)
- \( A/D \sim 1 \)

Varying around the static cylinder shedding frequency (\( f_{VS}^0 \))
- \( 0 < f_f/f_{VS}^0 < 2 \)

\( f_{VS}^0 = \text{vortex shedding frequency that would be observed with cylinder at rest} \)
(i.e. following the Strouhal relation)

Remember : \( Re \) has also a strong effect on the shedding process
→ Effect of the motion of the cylinder, for a given \( Re \) range.

We can observe: Changes in the flow pattern and in the shedding frequency
Oscillating cylinder: Patterns

- Limited range of forcing frequency and amplitude:
  - Two single vortices ("2S mode")
- Higher forcing frequencies and amplitudes:
  - Two pairs of vortices ("2P mode")
- Even higher frequencies and amplitudes: non-symmetric "P+S mode"
Oscillating cylinder : Frequency

Around $f_f / f_{VS^0} \sim 1$ → No change in pattern
→ But effect on the shedding frequency $f_{VS}$

i.e. $f_{VS}$ does not follow anymore the Strouhal relation

For a given Reynolds number and amplitude of imposed motion (A)

$f_f$ is increased
$f_{VS}$ is measured

Around $f_f / f_{VS^0} \sim 1$ → $f_{VS} = f_f$

= Lock-in phenomenon (wake capture)

!! Different from lock-in in VIV !!
Oscillating cylinder: Frequency

Lock-in range mainly depends on the imposed amplitude (ratio $A / D$)

$A/D \rightarrow \rightarrow \text{Range} \rightarrow \rightarrow$

![Graph showing the relationship between $A/D$ and $f/f_0^0$.]
Oscillating cylinder : Lift force

Imposed motion: \( y(t) = A \sin(2\pi f_f \ t) \)

→ The resulting lift force \( L(t) \) is not necessarily harmonic
   (idem static cylinder)

But through a standard Fourier analysis, it can approximated by

\[
L(t) = L_0 \sin(2\pi f_f \ t + \varphi) = \text{component of lift @ the forcing frequency (}f_f)\]

Driving parameters: \( A/D \) and \( f_f / f_{VS}^0 \) \( \rightarrow \) \( L_0(A/D, f_f / f_{VS}^0) \) and \( \varphi(A/D, f_f / f_{VS}^0) \)

Three different representations of the lift force:
1. Modulus and phase
2. Phased lift coefficients
3. Inertia and drag coefficients
Oscillating cylinder: Lift force

1. Modulus and phase
2. Phased lift coefficients
3. Inertia and drag coefficients

\[ C_L(t) = \bar{C}_L \sin(2\pi f f_t + \varphi) \]

→ both function of A/D and \( f_f / f_{VS}^0 \)

\[ \frac{\max(C_L)}{C_L(A=0)} \]

for \( f_f / f_{VS}^0 = 1 \)

adapted from Carberry et al. (2005)
Oscillating cylinder : Lift force

1. Modulus and phase
2. Phased lift coefficients
3. Inertia and drag coefficients

\[ y(t) = A \sin(2\pi f_f t) \]

\[ C_L(t) = (\bar{C}_L \cos \varphi) \sin(2\pi f_f t) + (\bar{C}_L \sin \varphi) \cos(2\pi f_f t) \]

\( \rightarrow \) both function of \( A/D \) and \( f_f / f_{VS}^0 \)

\( \bar{C}_L \cos \varphi \)

for \( A/D = 0.5 \)

\( \bar{C}_L \sin \varphi \)

\( \rightarrow \) in phase with \( y \) (and \( -\dot{y} \))

Conservative forces

In phase with \( \dot{y} \)

Non-conservative forces

\( \rightarrow \) might lead to positive or negative work

adapted from Sarpkaya (1979)
Oscillating cylinder : Lift force

1. Modulus and phase
2. Phased lift coefficients
3. Inertia and drag coefficients

\[ C_L(t) = -\rho \left( \frac{\pi D^2}{4} \right) C_M \dot{y} - \frac{1}{2} \rho D C_D \ddot{y} |\dot{y}| \]

- Inertia coefficient (added mass)
- Drag coefficient

Link with modulus: \( C_M = C_L \cos \phi \frac{U_r^2}{2\pi^3 (y_0/D)} \)

As expected for \( U_r \to 0 \), \( C_M \to 1 \) (motion in still fluid)

adapted from Sarpkaya (1979)
Vortex Induced Vibrations

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Free vibration: VIV

Consider a single dof system: a **simple linear oscillator**

![Diagram of a simple linear oscillator](image)

**VIV = frequency matching** between $f_{VS}$ and $f_S$

When?

**Strouhal law:**

$$St = \frac{f_{VS} D}{U_\infty} \Rightarrow f_{VS} = \frac{St U_\infty}{D}$$

**Matching:**

$$f_{VS} = \frac{St U_\infty}{D} = f_s \Rightarrow U_{VIV} = \frac{f_s D}{St}$$
Free vibration: VIV

Consider a single dof system: a simple linear oscillator

Remember from the dimensional analysis: \( \frac{A}{D} = F(U_r, \text{Re}, m_r, \eta_S) \)

In the case of free vibration, special interest for:

- \( f_S \): frequency of the free motion (‘S’ stands for Structural)
- \( f_S^0 \): frequency of the free motion (in still fluid, i.e. \( U_\infty = 0 \))
- \( A \): amplitude of the free motion
- \( f_{VS} \): frequency of the vortex shedding process (with motion)

\[
\begin{bmatrix}
\frac{A}{D}, & f_S, & f_{VS}
\end{bmatrix} = F(U_r, \text{Re}, m_r, \eta_S) \quad \text{with} \quad U_r = \frac{U_\infty}{f_S^0 D} \quad \text{\( f_S^0 \) independent of \( U_\infty \) (but not of the fluid properties)}
\]
Free vibration: VIV

Fixed structural parameters : \( D, f_S^0, m_r, \) and \( \eta \)

Fixed fluid properties : \( \nu, \rho \)

\[
\left[ \frac{A}{D}, \frac{f_S}{f_S^0}, \frac{f_{VS}}{f_S^0} \right] = F(U_r, Re, m_r, \eta_S)
\]

Airspeed effect \( (U_\infty) \)

For small variations of \( U_\infty \) in a Re range

\( \rightarrow \) Neglecting the Re effect

\( \rightarrow U_r \) only

\[
U_{VIV} = \frac{f_S D}{St} \rightarrow U_{VIV}^r = \frac{U_{VIV}}{f_S D} = \frac{1}{St}
\]
Free vibration: VIV

$A_{\text{max}}$

Lock-in range

in WATER

in AIR

FSI2011
Free vibration: VIV

Two key quantities to characterize VIV (practical interest):

- \( A_{\text{max}} \): max amplitude of the y motion, reached at \( U_r = 1/St \)
- Lock-in range: also occurs around \( U_r = 1/St \)

From the dimensional analysis:

\[
\begin{bmatrix} A, f_s, f_{VS} \end{bmatrix} = F(U_r, \text{Re}, m_r, \eta_S)
\]

\[
\rightarrow \begin{bmatrix} A_{\text{max}}/D, \text{Lockin}(U_r) \end{bmatrix} = F(\text{Re}, m_r, \eta_S)
\]

- Neglecting the Re effect
- \( m_r \) and \( \eta_S \) only
- Scruton number (\( \sim \) reduced damping)

\[
Sc = \frac{\pi}{2} (1 + m_r)\eta_S
\]

- Skop-Griffin number:

\[
SG = 4\pi^2 St^2 Sc
\]
Free vibration: VIV

\[ A_{\text{max}} = \text{Function}(SG) \]
\[ \text{Lock-in range} = \text{Function}(SG) \]

\( \rightarrow A_{\text{max}} < 1 \) or 2 diameters
(even for very low mass-damping combination)

\( \rightarrow A_{\text{max}} \approx 0 \) for heavy/damped structures

\( \rightarrow \text{Lock-in range} \uparrow \text{if } SG \downarrow \)

\( \rightarrow \text{VIV} = \text{Self-limited phenomenon} \)
Vortex Induced Vibrations

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Lock-in mechanism
Objective of the model:

- Estimation of $A_{\text{max}}$ and Lock-in range in a given Re range
  (Critical airspeed from the St number (from literature or wind tunnel tests))

- “Empirical” models, i.e. based on static and oscillating experimental data
- Many applications of VIV (in different fluids) → Tons of models (1960’s)
- In this course: 2D models only
- Classification:

  $F_{\text{Fluid}} = -m_A \ddot{y} + F$

  Inviscid added mass $m_A = \frac{\rho \pi D^2}{4}$

  Total force – added mass effect

Type A: forced system

Type B: fluid elastic

Type C: wake oscillator
Modelling VIV

Type A: Forced system models

From static cylinder:

\[ C_L(t) = \sqrt{2} C_L^{rms} \sin\left(\frac{St \ 2\pi U_\infty}{D} t\right) \]

\[ F(t) = L(t) = \frac{1}{2} \rho U_\infty^2 D \sqrt{2} C_L^{rms} \sin\left(\frac{St \ 2\pi U_\infty}{D} t\right) \]

\[ (m_s + m_A) \ddot{y} + 2\eta_s (m_s + m_A) \omega_s^0 \dot{y} + (m_s + m_A) \left(\frac{\omega_s^0}{\omega_s}\right)^2 y = L(t) \]

= Classical linear vibration problem

\[ \rightarrow \text{At resonance (} U_r = 1/\text{St}) : f_{VS} = f_s^0 \]

- Amplitude of motion inversely proportional to damping (\( \zeta \))
- Phase jump from 0 to \( \pi \)
- \( f_{VS} \) follows the Strouhal relation
Modelling VIV

**Type A: Forced system models**

\[
\left( m_S + m_A \right) \ddot{y} + 2\eta_S \left( m_S + m_A \right) \omega_S^0 \dot{y} + \left( m_S + m_A \right) \left( \omega_S^0 \right)^2 y = L(t)
\]

\[
\frac{y(t)}{D} = \frac{1}{2\pi^3} \frac{C_L U_r^2}{\left( 1 + m^* \right) \left( 1 - U_r^2 S^2 \right)^2 + 4\eta_S^2 U_r^2 S^2} \frac{\sin \left( 2\pi S t \frac{U}{D} + \varphi \right)}{\left( 1 - U_r^2 S^2 \right)^{1/2}}
\]

at resonance

\[
\frac{A}{D} = \frac{C_L}{4\pi^3 \left( 1 + m^* \right) S^2 \eta_S^2} = \frac{C_L}{2SG}
\]

**OK @ large SG numbers**

**KO @ low SG numbers**

No prediction of Lock-in

---

Type A

\[ U_\infty \]

\[ \rightarrow \]

\[ y \]

\[ \leftarrow \]

\[ F(t) \]

\[ \text{WAKE} \]

FIV2011

with \( C_L = 0.35 \)
Modelling VIV

Type B: Fluid elastic models

→ Force dependent of motion
(Amplitude, displacement, velocity, acceleration, combination of them …)

Similar to galloping models

→ Several types exist:

1. **Modified forcing model** (Blevins, 1990): \( L(t) = \frac{1}{2} \rho U_\infty^2 D C_L \left( \frac{A}{D} \right) \sin \left( \frac{St}{2\pi U_\infty} t \right) \)

\[
\frac{\max(C_L)}{C_L(A=0)}
\]

for \( f_f / f_S^0 = 1 \)

From oscillating cylinder (forced motion)

\[
\rightarrow C_L \left( \frac{A}{D} \right) = C_L^0 + \alpha \left( \frac{A}{D} \right) + \beta \left( \frac{A}{D} \right)^2
\]

\( C_L^0, \alpha \) and \( \beta \) from experiments
Modelling VIV

\[ L(t) = \frac{1}{2} \rho U^2 D C_L \left( \frac{A}{D} \right) \sin \left( \frac{St 2\pi U_\infty}{D} t \right) \]

\[ (m_S + m_A) \ddot{y} + 2\eta_S (m_S + m_A) \omega_S^0 \dot{y} + (m_S + m_A) (\omega_S^0)^2 y = L(t) \]

at resonance
(same than Type A)

\[ \frac{A}{D} = C_L \left( \frac{A}{D} \right) = \frac{C_L}{2SG} \quad C_L \left( \frac{A}{D} \right) = C_L^0 + \alpha \left( \frac{A}{D} \right) + \beta \left( \frac{A}{D} \right)^2 \]

\[ \rightarrow C_L^0 = (\alpha - 2SG) \left( \frac{A}{D} \right) + \beta \left( \frac{A}{D} \right)^2 \]

2\textsuperscript{nd} order equation to solve for A/D \rightarrow

OK @ large SG numbers
Better than Type A @ low SG numbers
(underestimate !)

Still no prediction of Lock-in
Modelling VIV

2. **Advanced forcing models:**
   - Amplitude and phase of the force depend on amplitude and reduced frequency of the motion
   - Concept of phased lift coefficients (oscillating cylinder)

   \[ L\left(\frac{A}{D}, U_r, t\right) = \frac{1}{2} \rho U_\infty^2 \left[ \bar{C}_L \cos \varphi \left( \frac{A}{D}, U_r \right) \sin(\omega t) + \bar{C}_L \sin \varphi \left( \frac{A}{D}, U_r \right) \cos(\omega t) \right] \]

   Identified from imposed motion \( \Rightarrow \) \( U_r = U_\infty / (fD) \)

   \[ m_S \ddot{y} + 2\eta_S m_S \omega_S^0 \dot{y} + m_S \left( \omega_S^0 \right)^2 y = L\left(\frac{A}{D}, U_r, t\right) \]

   is valid outside coincidence

   \[ \text{At coincidence :} \quad \frac{A}{D} = -\frac{\bar{C}_L \sin \varphi (A / D, U_r = 1 / St)}{2SG} \]
Modelling VIV

1. Modified forcing model (Blevins, 1990):

\[ L(t) = \frac{1}{2} \rho U^2 \frac{D}{L} \sin \left( \frac{St}{D} t \right) \]

1. Advanced forcing models:

\[ L\left( \frac{A}{D}, U_r, t \right) = \frac{1}{2} \rho U^2 \left[ \bar{C}_L \cos \varphi \left( \frac{A}{D}, U_r \right) \sin (\omega t) + \bar{C}_L \sin \varphi \left( \frac{A}{D}, U_r \right) \cos (\omega t) \right] \]

2. Time domain fluidelastic models:

→ Explicit use of the time dependence of time (Chen et al. 1995)

\[ L\left( \frac{A}{D}, U_r, y, \dot{y}, \ddot{y} \right) = -m \left( \frac{A}{D}, U_r \right) \ddot{y} - \rho U^2 c \left( \frac{A}{D}, U_r \right) y - \rho U^2 k \left( \frac{A}{D}, U_r \right) \dot{y} + \frac{1}{2} \rho U^2 C_L \sin (\omega_{vS} t) \]

Classical fluidelastic forces (added mass, damping and stiffness)

forcing term, at the frequency of vortex shedding
Modelling VIV

\[ L \left( \frac{A}{D}, U_r, y, \dot{y}, \ddot{y} \right) = \left[ -m \left( \frac{A}{D}, U_r \right) \ddot{y} - \rho U_\infty^2 c \left( \frac{A}{D}, U_r \right) \dot{y} - \rho U_\infty^2 k \left( \frac{A}{D}, U_r \right) y \right] + \frac{1}{2} \rho U_\infty^2 C_L \sin(\omega_{Vs} t) \]

Problem of the Chen’s model: harmonic motion assumed, because of the dependence of \( m, c \) and \( k \) on \( U_r \) and \( A/D \) (identification based on oscillating cylinder experiments)

Another (much more complex) model, proposed by Simiu & Scanlan (1996)

\[ L(y, \dot{y}, t) = \frac{1}{2} \rho U_\infty^2 D \left[ c(U_r) \left( 1 - \epsilon \frac{y^2}{D^2} \right) \dot{y} + k(U_r) \frac{y}{U_\infty} + C_L(U_r) \sin(\omega_{Vs} t) \right] \]

depends on \( U_r \) only \( \rightarrow \) not especially harmonic

non linear term to account for amplitude effect
Modelling VIV

Type A: forced system

\[ F(t) = \frac{1}{2} \rho U_\infty^2 D \sqrt{2} C_{L^{rms}} \sin \left( \frac{St \ 2\pi U_\infty}{D} t \right) \]

Type B: Fluid elastic

\[ F(t) = \frac{1}{2} \rho U_\infty^2 D C_L \left( \frac{A}{D} \right) \sin \left( \frac{St \ 2\pi U_\infty}{D} t \right) \]

\[ F \left( \frac{A}{D}, U_r, t \right) = \frac{1}{2} \rho U_\infty^2 \left[ \bar{C}_L \cos \varphi \left( \frac{A}{D}, U_r \right) \sin (\omega t) + \bar{C}_L \sin \varphi \left( \frac{A}{D}, U_r \right) \cos (\omega t) \right] \]

\[ F \left( \frac{A}{D}, U_r, y, \dot{y}, \ddot{y} \right) = -m \left( \frac{A}{D}, U_r \right) \ddot{y} - \rho U_\infty^2 c \left( \frac{A}{D}, U_r \right) \dot{y} - \rho U_\infty^2 k \left( \frac{A}{D}, U_r \right) y + \frac{1}{2} \rho U_\infty^2 C_L \sin (\omega_{VS} t) \]

\[ F(p, \dot{y}, t) = \frac{1}{2} \rho U_\infty^2 D \left[ c(U_r) \left( 1 - \epsilon \frac{y^2}{D^2} \right) \frac{\dot{y}}{U_\infty} + k(U_r) \frac{y}{U_\infty} + C_L(U_r) \sin (\omega_{VS} t) \right] \]
Limitations of Types A and B:

1. Limited to harmonic motions (except at the price of cumbersome models (Simiu & Scanlan 1996))
   - Difficult to generalize to more complex motions (multifrequencies …)
   - Not a major limitation, because VIV often results in quasi-harmonic motions

2. More important: no physical interpretation of wake as a global mode
"Fluid force is the result of the wake dynamics, itself influenced by the cylinder motion"

→ Coupling of two systems: CYLINDER (y variable) and WAKE (q variable)

\[
\begin{align*}
    m\ddot{y} + ... &= F(q) \\
    W[q(t)] &= G(y,...)
\end{align*}
\]

F(q) = effect of the wake on the cylinder
G(y) = effect of the cylinder on the wake
W[q(t)] = dynamics of the free wake

Several formulations exist for F(q), G(y), W[q(t)] and the choice for q(t)
In all cases, the parameters of the models come from experiments on:

- Static cylinder → Lift force → F(q)
  Wake dynamics → W[q(t)]
- Oscillating cylinder → Wake dynamics → G(y)
Modelling VIV

Type C: Wake oscillator models

\[
\begin{align*}
    m\ddot{y} + \cdots &= F(q) \\
    W[q(t)] &= G(y,\ldots)
\end{align*}
\]

Most natural choice of \( q(t) \) by Hartlen and Currie (pioneers): \( q(t) = C_L(t) \)

\( \to \) Directly: \( F(q(t)) = F(C_L(t)) = \frac{1}{2} \rho U_\infty^2 DC_L(t) \)

\( W[q(t)] \) must respect two essential features of the wake as a global mode:

1. The oscillating wake results from a self-sustained flow instability, as linear unstable mode
2. This instability is self-limited, as a nonlinear global mode

A Rayleigh equation capture these features: \( W[q(t)] = \ddot{q} - a\dot{q} + b\dot{q}^3 + \omega^2 q = 0 \)

(a van der Pol oscillator does it too) with \( a, b > 0 \)

Finally, \( G(y) = c\dot{y} \) (rather arbitrary choice by H&C)
Modelling VIV

Type C : Wake oscillator model by Facchinetti, de Langre & Biolley

- Using the wake dof : \( q(t) = 2C_L(t) / C_L^0 \)
- Based on a van der Pol oscillator for \( W[q(t)] \)
- Using acceleration for coupling the wake to the structure

\[
\begin{align*}
\ddot{y} + \left( m_s + \rho D^2 \frac{\pi}{4} \right) \dot{y} + \left( c_s + \frac{1}{2} \rho U_\infty D \ St \ C_D \right) \dot{y} + k_s y &= \frac{1}{4} \rho U_\infty^2 D C_L^0 q \\
\dot{q} + 2\pi St \frac{U_\infty}{D} \epsilon (q^2 - 1) \dot{q} + \left( 2\pi St \frac{U_\infty}{D} \right)^2 &= \frac{A}{D} \ddot{y}
\end{align*}
\]

from force measurement in static cylinder experiments (typical values: \( C_L^0 = 0.3 \) and \( C_D = 1.2 \))

from wake analysis (hotwire) in static cylinder exp. (typical values : \( St = 0.2 \) and \( \epsilon = 0.3 \))

from wake analysis in oscillating cylinder exp. (typical value : \( A = 12 \))
Modelling VIV

Type C: Wake oscillator model by Facchinetti, de Langre & Biolley

Using typical values ($+C_D=1$)

➔ Strong underestimation of the amplitude

➔ But good qualitative influence of the SG parameter on the lock-in range

![Diagram](image_url)
Modelling VIV

Type C: Wake oscillator model by Y. Tamura and G. Matsui (1979)

\[
\begin{align*}
\dot{Y} + \left[ 2\eta_s + m_r \left( f + C_D \right) \frac{\nu}{2\pi St} \right] \dot{Y} + Y &= -f \frac{m_r \nu^2}{(2\pi St)^2} \\
\ddot{\alpha} - 2\xi \nu \left[ 1 - \frac{4f^2}{C_{L,0}^2} \alpha^2 \right] \dot{\alpha} + \nu^2 \alpha &= -m_r \dot{Y} - 2\pi St \nu \dot{Y} \\
C_L &= -f \left( \alpha + 2\pi St \frac{\dot{Y}}{\nu} \right)
\end{align*}
\]

Y = non-dim vertical displacement
\(\alpha = \) wake position

from force measurement in static cylinder experiments (typical values: \(C_L^0=0.3\) and \(C_D=1.2\))
from wake analysis (hotwire) in static cylinder exp. (typical values: \(St=0.2\) and \(f = 1.16\))
from wake analysis in oscillating cylinder exp.
Vortex Induced Vibrations

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Lock-in mechanism
Lock-in mechanism

→ Link between Lock-in and linear coupled-mode flutter (proposed by de Langre 2006)

Wake oscillator model by Facchinetti, de Langre & Biolley (FLB)

\[
\begin{align*}
\dot{q} + 2\pi St\frac{U_\infty}{D} \varepsilon (q^2 - 1) \dot{q} + (2\pi St \frac{U_\infty}{D})^2 &= \frac{A}{D} \ddot{y} \\
\end{align*}
\]

Non-dimensionalized:

\[
\begin{align*}
\ddot{Y} + \lambda \dot{Y} + Y &= M \Omega^2 q \\
\dot{\epsilon} \Omega (q^2 - 1) \dot{Y} + \Omega^2 q &= \lambda Y
\end{align*}
\]

Neglecting damping and NL terms: (i.e. not responsible for lock-in)

\[
\begin{align*}
\ddot{Y} + Y &= M \Omega^2 q \\
\dot{Y} + \Omega^2 q &= \lambda Y
\end{align*}
\]

→ \( \Omega \) is the only driving parameter  \( \Omega = St \ U_r = \frac{f_{VS}}{f} \)
Solution of this system is straightforward: \( (Y, q) = (Y_0, q_0) e^{i\omega t} \)

\[
\begin{align*}
\ddot{Y} + Y &= M\Omega^2 q \\
\ddot{q} + \Omega^2 q &= A\dot{Y}
\end{align*}
\]

Feeding the solution into the system equations
→ frequency equation: \( D(\omega) = \omega^4 + [(AM - 1) \Omega^2 - 1] \omega^2 + \Omega^2 = 0 \)

if \( AM < 1 \) (verified in practice)
→ Two modal frequencies \( \omega_1 \) and \( \omega_2 \)

\[
\begin{align*}
\omega_1^2 &= \frac{1}{2} \left[ 1 + (1 - AM) \Omega^2 + \sqrt{(1 + (1 - AM) \Omega^2)^2 - 4 \Omega^2} \right] \\
\omega_2^2 &= \frac{1}{2} \left[ 1 + (1 - AM) \Omega^2 - \sqrt{(1 + (1 - AM) \Omega^2)^2 - 4 \Omega^2} \right]
\end{align*}
\]

Variation of \( \omega_1 \) and \( \omega_2 \) with \( \Omega \) (= driving parameter) ?
Lock-in mechanism

Depending on the sign of the radicand:

\[
\omega_{1,2}^2 = \frac{1}{2} \left[ 1 + (1 - AM) \Omega^2 \pm \sqrt{(1 + (1 - AM) \Omega^2)^2 - 4\Omega^2} \right]
\]

\[
(1 + (1 - AM) \Omega^2)^2 - 4\Omega^2 \geq 0 \quad \iff \quad \Omega < \frac{1}{1 + \sqrt{AM}} \text{ or } \Omega > \frac{1}{1 - \sqrt{AM}}
\]

→ \( \omega_1 \) and \( \omega_2 \) are real modal frequencies
→ two neutrally stable modes co-exist
  (neutrally because damping is neglected here)

The corresponding mode shapes:

\[
\frac{Y_0}{q_0} = \frac{M \Omega^2}{1 - \omega^2} = \frac{\Omega^2 - \omega^2}{A\omega^2}
\]

→ close to \( \omega_R = \Omega \) = Wake mode
→ close to \( \omega_R = 1 \) = Structural mode
**Lock-in mechanism**

In the range \( \frac{1}{1 + \sqrt{AM}} < \Omega < \frac{1}{1 - \sqrt{AM}} \)

Two modes exist, but with complex conjugate frequencies

\[
\omega_{1,2} = \sqrt{\frac{\Omega}{1 + \tan^2 \theta}} \left( 1 \pm i \tan \theta \right)
\]

\[
\theta = \frac{1}{2} \tan^{-1} \sqrt{\frac{4\Omega^2 - (1 + (1 - AM)\Omega^2)^2}{1 + (1 - AM)\Omega^2}}
\]

→ **Coupled-Mode Flutter solution (CMF)**

= merging of two frequencies

of the neutral modes

→ Range of instability

one stable mode and one unstable mode
Lock-in mechanism

Coupled-Mode Flutter solution (CMF)

one **stable** mode and one **unstable** mode

\[ \omega_i < 0 \quad \omega_i > 0 \]

In the CMF range:

- No distinction between modes
- Strong deviation of the frequency of the wake mode from \( \omega = \Omega \) (Strouhal relation), instead \( \omega \sim 1 \)
- One unstable mode instead of two well separated stable (pure wake and pure structural) modes

\[ \rightarrow \] **CMF range ~ Lock-in range**
Lock-in mechanism

Most unstable mode?

Remember we assumed no damping: $\varepsilon = \lambda = 0$

Out of the Lock-in range, it is interesting to know which mode is dominant

Considering non-zero damping: $\varepsilon$ and $\lambda > 0$

A first order expansion of the frequency equation yields:

$$\omega = \omega_0 + i\varepsilon \left[ \Omega \omega_0^2 \frac{1 - \omega_0^2}{2(\omega_0^4 - \Omega^2)} \right] - i\lambda \left[ \omega_0^2 \frac{\Omega^2 - \omega_0^2}{2(\omega_0^4 - \Omega^2)} \right]$$

if $\omega_0$ stands for $\omega_W \rightarrow \varepsilon$ and $\lambda$ have a destabilization effect on the wake mode

if $\omega_0$ stands for $\omega_S \rightarrow \varepsilon$ and $\lambda$ have a damping effect on the structural mode

$\rightarrow$ in both cases the most unstable mode has the frequency of the wake mode
Lock-in mechanism

\[ \text{Resulting frequency of oscillation} = \text{one of the dominant mode} \]

i.e. the wake mode

\[ \text{AM} = 0.05 \quad \text{AM} = 0.75 \]

Small Lock-in range
Small deviation from Strouhal

Large Lock-in range
Large deviation from Strouhal outside CMF

de Langre 2006
Lock-in mechanism

Critical mass ratio

Lock-in range = limit of CMF range:

\[
\frac{1}{1 + \sqrt{AM}} < \Omega < \frac{1}{1 - \sqrt{AM}}
\]

The extend of Lock-in is significantly affected by the ratio of the cylinder mass to the fluid mass:

\[m^* = \frac{m_s}{\rho \pi D^2 / 4} \neq \frac{m_s + m_F}{\rho D^2} \]

\[m^* = \frac{4}{\pi} \mu - 1\]

The lower Lock-in limit:

\[\Omega^{MIN} = \frac{1}{1 + \sqrt{AM}} \rightarrow St \ U_r^{MIN} = \left[1 + \sqrt{\frac{AC_L^0}{4\pi^2 St^2 (m^* + 1)}}\right]^{-1}\]

The upper Lock-in limit:

\[\Omega^{MAX} = \frac{1}{1 - \sqrt{AM}} \rightarrow St \ U_r^{MAX} = \left[1 - \sqrt{\frac{AC_L^0}{4\pi^2 St^2 (m^* + 1)}}\right]^{-1}\]
Lock-in mechanism

Critical mass ratio

The lower Lock-in limit:

\[ U_r^{MIN} = \frac{1}{St} \left[ 1 + \frac{AC_L^0}{4\pi^3 St^2 (m^* + 1)} \right]^{-1} \]

The upper Lock-in limit:

\[ U_r^{MAX} = \frac{1}{St} \left[ 1 - \frac{AC_L^0}{4\pi^3 St^2 (m^* + 1)} \right]^{-1} \]

Depending on St, m* and C_L^0 only

\[ m_{CRIT} = \frac{AC_L^0}{4\pi^3 St^2} - 1 \]

\( m_{CRIT} \) depends on St, m* and C_L^0 only.

\[ U_R(U_L) \] Depend on St, m* and C_L^0 only

\( m_{CRIT} \) depends on St, m* and C_L^0 only

→ Depending on the value of C_L^0, a critical value of m* exists for which lock-in persists forever
Vortex Induced Vibrations

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Summary

- **Static cylinder** → strong dependence on Re
- **Oscillating cylinder**
  
  Strong dependence on $f/f_{VS}^{0}$ forcing frequency
  
  A/D forcing amplitude

→ Effects on the flow pattern (2S, 2P S+P modes) & vortex shedding frequency

→ lock-in

Three different representations of the lift force:

- Modulus and phase
- Phased lift coefficients
- Inertia and drag coefficients

→ function of A/D and $f_{f}/f_{VS}^{0}$
Summary

- Free cylinder $\rightarrow$ VIV
  \[ \frac{A}{D}, \frac{f_s}{f_s^0}, \frac{f_{VS}}{f_s^0} = F(U_r, \text{Re}, m_r, \eta_S) \]

  $A_{\text{max}} = \text{Function}$(SG)

  Lock-in range = Function(SG)

  $\rightarrow$ VIV = Self-limited phenomenon

$\rightarrow$ 3 types of models:
- forced system models
- fluid elastic models
- wake oscillator models
Summary

Type A: forced system

\[ U_\infty \rightarrow y \rightarrow F(t) \rightarrow \text{WAKE} \]

\[ F(t) = \frac{1}{2} \rho U_\infty^2 D \sqrt{2C_{L_{rms}}} \sin \left( \frac{St \ 2\pi U_\infty t}{D} \right) \]

Type B: Fluid elastic

\[ U_\infty \rightarrow y \rightarrow F(y(t),t) \rightarrow \text{WAKE} \]

Type C: wake oscillator

\[ U_\infty \rightarrow y \rightarrow G(y) \rightarrow \text{WAKE} \]

\[ m\ddot{y} + \ldots = F(q) \]

\[ W[q(t)] = G(y,\ldots) \]

Type A B and C are empirical models.

Type C models the cause of the fluid forces (that types A & B take for granted).

\[ A/D \]

\[ 1.5 \]

\[ 1 \]

\[ 0.5 \]

\[ 0 \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ 10^0 \]

\[ 10 \]

\[ \text{modified forcing model} \]

\[ \text{forcing model} \]

\[ \text{wake oscillator} \]
Summary

- Lock-in mechanism through linear analysis
  \[
  \begin{align*}
  \ddot{Y} + Y &= M\Omega^2 q \\
  \dot{q} + \Omega^2 q &= A\ddot{Y}
  \end{align*}
  \]
  → Coupled-Mode Flutter (CMF)

- Critical mass ratio → infinite lock-in range

- Link between Airfoil flutter and Lock-in
  pitch \((\alpha)\) and plunge \((h)\)
  transverse displacement \((y)\) and wake variation

\[
\begin{align*}
\begin{pmatrix}
  m_s & S \\
  S & I_{\alpha}
\end{pmatrix}
\begin{pmatrix}
  \dot{h} \\
  \dot{\alpha}
\end{pmatrix} + \rho \pi \phi^2
\begin{pmatrix}
  1 & 1 \\
  \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
  \ddot{h} \\
  \ddot{\alpha}
\end{pmatrix} + \rho U_c \pi
\begin{pmatrix}
  1 & \frac{1}{2} \\
  \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
  \dot{h} \\
  \dot{\alpha}
\end{pmatrix} + \frac{1}{2} \rho U_c^2 \pi
\begin{pmatrix}
  0 & 0 \\
  0 & -e_c
\end{pmatrix}
\begin{pmatrix}
  k \\
  \alpha
\end{pmatrix} = \begin{pmatrix}
  0 \\
  0
\end{pmatrix}
\end{align*}
\]

related to position of the separation point

\[
C_L(t)
\]
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