Aeroelasticity and Experimental Aerodynamics

Lecture 1:
Introduction – Equations of motion
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Introduction

- Aerelasticity is the study of the interaction of inertial, structural and aerodynamic forces on aircraft, buildings, surface vehicles etc.
Why is it important?

• The interaction between these three forces can cause several undesirable phenomena:
  – Divergence (static aeroelastic phenomenon)
  – Flutter (dynamic aeroelastic phenomenon)
  – Vortex-induced vibration, buffeting (unsteady aerodynamic phenomena)
  – Limit Cycle Oscillations (nonlinear aeroelastic phenomenon)
Static Divergence

NASA wind tunnel experiment on a forward swept wing
Flutter experiment: Winglet under fuselage of a F-16. Slow Mach number increase.

The point of this experiment was to predict the flutter Mach number from subcritical test data and to stop the test before flutter occurs.
Vortex-induced vibrations

Flow visualization of vortices shed behind a cylinder

Vortex-induced vibrations
Limit Cycle Oscillations

Stall flutter of a wing at an angle of attack

Torsional LCO of a rectangle
Even more LCOs

Galloping of a bridge deck

Torsional oscillations of a bridge deck
Many more LCOs
These phenomena do not occur only in the lab

Tacoma Narrows Bridge Flutter

Glider Limit Cycle Oscillations

Various phenomena
Even on very expensive kit

- Store-induced LCO on F-16
- Fin buffeting on F-18
How to avoid these phenomena?

- Aeroelastic Design (Divergence, Flutter, Control Reversal)
- Wind tunnel testing (Aeroelastic scaling)
- Ground Vibration Testing (Complete modal analysis of aircraft structure)
- Flight Flutter Testing (Demonstrate that flight envelope is flutter free)
Wind Tunnel Testing

¼ scale F-16 flutter model

F-22 buffet Test model
Ground Vibration Testing

GVT of F-35 aircraft

GVT of A340

Space Shuttle horizontal GVT
Flight Flutter Testing

- Real-time frequency analyzer
- Spectral analysis facility ground station
- Stability trends

Data flow:
- Data from flight testing
- Data from real-time frequency analyzer
- Data from spectral analysis facility ground station
- Stability trends from data analysis
So what is in the course?

- Introduction to aeroelastic modeling
- Modeling of static aeroelastic issues and phenomena:
  - Divergence
- Modeling of dynamic aeroelastic phenomena:
  - Flutter, vortex-induced vibration, stall flutter, galloping
- Practical aeroelasticity:
  - Aeroelastic design
  - Ground Vibration Testing, Flight Flutter Testing
- Non-aircraft aeroelasticity
A bit of history

• The first ever flutter incident occurred on the Handley Page O/400 bomber in 1916 in the UK.
• A fuselage torsion mode coupled with an antisymmetric elevator mode (the elevators were independently actuated)
• The problem was solved by coupling the elevators
More history

- Control surface flutter became a frequent phenomenon during World War I and in the interwar period.
- It was solved in the mid-twenties by mass balancing the control surface.
Increasing airspeed

- Aircraft flight speeds increased significantly during the 20s and 30s.
- A number of high-speed racing aircraft suffered from flutter problems.

Supermarine S-4
Verville-Sperry R-3
Loening R-4
Curtiss R-6
US flutter experiences in the 1930s

General Aviation YO-27: Wing-aileron and rudder-fuselage flutter

Fairchild F-24: Wing-aileron and tail flutter

Boeing YB-9A: Rudder-fuselage LCO

Curtiss YA-8: Rudder-fin flutter
Other historic examples

- Aircraft that experienced aeroelastic phenomena
  - Handley Page O/400 (elevators-fuselage)
  - Junkers JU90 (fluttered during flight flutter test)
  - P80, F100, F14 (transonic aileron buzz)
  - T46A (servo tab flutter)
  - F16, F18 (external stores LCO, buffeting)
  - F111 (external stores LCO)
  - F117, E-6 (vertical fin flutter)
F-117 crash

- A F-117 crashed during an airshow in Maryland in September 1997
- The inquest found that four fasteners that connected the elevon actuator to the wing structure were missing.
- This reduced the actuator-elevon stiffness, leading to elevon-wing flutter
Aeroelastic Modeling

- Aircraft are very complex structures with many modes of vibration and can exhibit very complex fluid-structure interaction phenomena.
- The exact modeling of the aeroelastic behaviour of an aircraft necessitates the coupled solution of:
  - The full compressible Navier Stokes equations
  - The full structural vibrations equations
- As this is very difficult, we begin with something simpler:
Pitch Plunge Airfoil

Two-dimensional, two degree-of freedom airfoil, quite capable of demonstrating most aeroelastic phenomena.

\[ \alpha = \text{pitch degree of freedom} \]
\[ h = \text{plunge degree of freedom} \]
\[ x_f = \text{position of flexural axis (pivot)} \]
\[ x_c = \text{position of centre of mass} \]
\[ K_h = \text{plunge spring stiffness} \]
\[ K_\alpha = \text{pitch spring stiffness} \]

In fact, we will simplify even further and consider a flat plate airfoil (no thickness, no camber)
There are two aspects to each aeroelastic models
   - A structural model
   - An aerodynamic model
In some cases a control model is added to represent the effects of actuators and other control elements
Develop the structural model
Structural Model Details

- Use total energy conservation

\[ dT = \frac{1}{2} dm \left( \dot{h} + (x - x_f) \dot{\alpha} \right)^2 \]
Kinetic Energy

- The total kinetic energy is given by

\[ T = \frac{1}{2} m \dot{h}^2 + S \dot{h} \dot{\alpha} + \frac{1}{2} I_{\alpha} \dot{\alpha}^2 \]

where

\[ S = m (c/2 - x_f) \]

\[ I_{\alpha} = \frac{1}{3} m \left( c^2 - 3c x_f + 3x_f^2 \right) \]
Potential Energy

• The potential energy is simply the energy stored in the two springs, i.e.

\[ V = \frac{1}{2} K_h h^2 + \frac{1}{2} K_\alpha \alpha^2 \]

• Notice that gravity can be conveniently ignored

• Total energy=kinetic energy+potential energy
Equations of motion (1)

• The equations of motion can be obtained by inserting the expression for the total energy into Lagrange’s equation

\[
\frac{\partial}{\partial t} \left( \frac{dT}{d\dot{q}} \right) + \frac{dV}{dq} = 0
\]

\[
q = [h \quad \alpha]^T
\]
Equations of motion (2)

- This should yield a set of two equations of the form

\[
\begin{pmatrix}
    m & S \\
    S & I_\alpha
\end{pmatrix}
\begin{Bmatrix}
    \ddot{h} \\
    \ddot{\alpha}
\end{Bmatrix}
+ \begin{pmatrix}
    K_h & 0 \\
    0 & K_\alpha
\end{pmatrix}
\begin{Bmatrix}
    h \\
    \alpha
\end{Bmatrix}
= 0
\]

- or, \( M\ddot{q} + Kq = Q \)

where \( Q \) is a vector of external forces
The possible aerodynamic models depend on flow regime and simplicity.

In general, only four flow regimes are considered by aeroelasticians:
- Incompressible
- Subsonic
- Transonic
- Supersonic

For the moment we will deal only with incompressible modeling.
Incompressible, Unsteady Aerodynamics

Oscillating airfoils leave behind them a strong vortex street. The vorticity in the wake affects the flow over the airfoil:

*The instantaneous aerodynamic forces depend not only on the instantaneous position of the airfoil but also on the position and strength of the wake vortices.*

This means that instantaneous aerodynamic forces depend not only on the current motion of the airfoil but on all its motion history from the beginning of the motion.
Wake examples (Pitch)

Pitching airfoil - Low frequency

Pitching airfoil - High frequency
Wake examples (Plunge)

Plunging airfoil-
Low amplitude

Plunging airfoil-
High amplitude
Quasi-steady aerodynamics

- The simplest possible modeling consists of ignoring the effect of the wake
- Quasi-steady models assume that there are only four contributions to the aerodynamic forces:
  - Horizontal airspeed $U$, at angle $\alpha(t)$ to airfoil
  - Airfoil plunge speed, $\dot{h}(t)$
  - Normal component of pitch speed, $(x_f - x) \dot{\alpha}(t)$
  - Local velocity induced by the vorticity around the airfoil, $v_i(x,t)$
The aerodynamic force acting on the wing is the lift and it is placed on the quarter chord (aerodynamic centre). There is also an aerodynamic moment acting around the flexural axis.

NB: The lift is defined positive downwards
Lift coefficient

- The airfoil is uncambered but the pitching motion causes an effective camber with slope \( \frac{dz}{dx} = (x_f - x) \frac{\dot{\alpha}}{U} \)

- From thin airfoil theory, \( c_l = 2\pi(A_0 + A_1/2) \), where

\[
A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta \quad A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n\theta d\theta
\]

\[
x = \frac{c}{2} (1 - \cos \theta)
\]
Lift coefficient (2)

- Substituting all this into the equation for the lift coefficient and carrying out the integrations we get

\[ c_l(t) = 2\pi \left( \alpha(t) + \frac{\dot{h}(t)}{U} + \left( \frac{3}{4} c - x_f \right) \frac{\dot{\alpha}(t)}{U} \right) \]

- This is the total circulatory lift acting on the airfoil. There is another type of lift acting on it, which will be presented in a bit.
Moment coefficient

- The moment coefficient around the leading edge (according to thin airfoil) theory is given by $c_m = -c_l/4 - \pi(A_1 - A_2)/4$

- Therefore, the moment coefficient around the flexural axis is given by
  $c_{mx_f} = c_m + x_f c_l/c$

- Substituting and integrating yields
  $$C_{m_{xf}}(t) = -\frac{c\pi}{8U} \dot{\alpha}(t) + \left(\frac{x_f}{c} - \frac{1}{4}\right) C_l(t)$$
Added Mass

- Apart from the circulatory lift and moment, the air exerts another force on the airfoil.
- The wing is forcing a mass of air around it to move. The air reacts and this force is known as the added mass effect.
- It can be seen as the effort required to move a cylinder of air with mass \( \pi \rho b^2 \) where \( b = c/2 \).
- This force causes both lift and moment contributions.
Full lift and moment

\[ l(t) = \rho \pi b^2 \left( \ddot{h} - \left(x_f - \frac{c}{2}\right) \dddot{\alpha} \right) + \rho \pi b^2 U \dot{\alpha} + \rho U^2 c \pi \left( \alpha(t) + \frac{\dot{h}(t)}{U} + \left(\frac{3}{4}c - x_f\right) \frac{\dot{\alpha}(t)}{U} \right) \]

\[ m_{xf}(t) = \rho \pi b^2 \left(x_f - \frac{c}{2}\right) \left( \ddot{h} - \left(x_f - \frac{c}{2}\right) \dddot{\alpha} \right) - \frac{\rho \pi b^4}{8} \dddot{\alpha} - \left(\frac{3}{4}c - x_f\right) \rho \pi b^2 U \ddot{\alpha} + \rho U^2 c^2 \pi \left( \alpha(t) + \frac{\dot{h}(t)}{U} + \left(\frac{3}{4}c - x_f\right) \frac{\dot{\alpha}(t)}{U} \right) - \frac{1}{16} \rho U c^3 \pi \dddot{\alpha} \]  

(1)

These are to be substituted into the structural equations of motion:

\[
\begin{pmatrix}
    m & S \\
    S & I_\alpha
\end{pmatrix} \begin{bmatrix}
    \ddot{h} \\
    \dddot{\alpha}
\end{bmatrix} + \begin{pmatrix}
    K_h & 0 \\
    0 & K_\alpha
\end{pmatrix} \begin{bmatrix}
    h \\
    \alpha
\end{bmatrix} = \begin{bmatrix}
    -l(t) + p(t) \\
    m_{xf}(t) + r(t)
\end{bmatrix}
\]
Full aeroelastic equations of motion

- The equations of motion are second order, linear, ordinary differential equations.
- Notice that the equations are of the form
  \[(A + \rho B)\ddot{q} + (C + \rho UD)\dot{q} + (E + \rho U^2 F)q = 0\]
- And that there are mass, damping and stiffness matrices both aerodynamic and structural
Pitch-plunge equations of motion

\[
\begin{pmatrix}
  m & S \\
  S & I_\alpha
\end{pmatrix}
\begin{Bmatrix}
  \ddot{h} \\
  \ddot{\alpha}
\end{Bmatrix}
+ \rho \pi b^2 \left( \begin{pmatrix}
  1 \\
  \frac{c}{2} - x_f
\end{pmatrix}
\begin{pmatrix}
  \frac{c}{2} - x_f \\
  \frac{c}{2} - x_f
\end{pmatrix}
\right)
\begin{Bmatrix}
  \dot{h} \\
  \dot{\alpha}
\end{Bmatrix}
\]

\[
\rho U c_\pi \left( \frac{1}{-ec} \left( \frac{3}{4} c - x_f \right) + \frac{c}{4} \right)
\begin{Bmatrix}
  \dot{h} \\
  \dot{\alpha}
\end{Bmatrix}
\]

\[
\begin{pmatrix}
  K_h & 0 \\
  0 & K_\alpha
\end{pmatrix}
\begin{Bmatrix}
  h \\
  \alpha
\end{Bmatrix}
+ \rho U^2 c_\pi \left( \begin{pmatrix}
  0 & 1 \\
  0 & -ec
\end{pmatrix}
\begin{Bmatrix}
  h \\
  \alpha
\end{Bmatrix}
\right) = \begin{Bmatrix}
  0 \\
  0
\end{Bmatrix}
\]

These are the full equations of motion for the pitch-plunge airfoil with quasi-steady aerodynamics. We will investigate them in more detail now.
Static Aeroelasticity

- First, we will study the static equilibrium of the system.
- Static means that all velocities and accelerations are zero.
- The equations of motion become

\[
\begin{pmatrix}
K_h & \rho U^2 c \pi \\
0 & K_\alpha - \rho U^2 e c^2 \pi
\end{pmatrix}
\begin{Bmatrix}
h \\
\alpha
\end{Bmatrix}
= \begin{Bmatrix}
0 \\
0
\end{Bmatrix}
\]
• Let us apply an external moment $M$ around the flexural axis
• The static equilibrium equations become

\[
\begin{pmatrix}
K_h & \frac{\rho U^2 c \pi}{K_\alpha - \rho U^2 ec^2 \pi} \\
0 & K_\alpha - \rho U^2 ec^2 \pi
\end{pmatrix}
\begin{pmatrix}
h \\
\alpha
\end{pmatrix}
=
\begin{pmatrix}
0 \\
M
\end{pmatrix}
\]

• The second equation can only be satisfied if

$$\alpha = \frac{M}{(K_\alpha - \rho U^2 ec^2 \pi)}$$

• Then, the first equation can only be satisfied if

$$h = -\frac{\rho U^2 c \pi M}{K_h(K_\alpha - \rho U^2 ec^2 \pi)}$$
Aerodynamic Coupling (2)

• This phenomenon is called aerodynamic coupling. Changing the pitch angle causes a change in the plunge.
• This is logical since increased pitch means increased lift.
• However, if we apply a force $F$ on the flexural axis, the equations become

$$\begin{pmatrix} K_h & \rho U^2 c \pi \\ 0 & K_\alpha - \rho U^2 e c^2 \pi \end{pmatrix} \begin{pmatrix} h \\ \alpha \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$
The second equation can only be satisfied if $\alpha=0$

The first equation then gives $h=F/K_h$

In this case, there is no aerodynamic coupling. Increasing the plunge does not affect the pitch.

This is not the general case. The pitch-plunge model ignores 3D aerodynamic effects

In real aircraft bending and torsion are both coupled.
Static Divergence (1)

• Look at the second static equilibrium equation with an applied moment
\[
(K_{\alpha} - \rho U^2 ec^2\pi) \alpha = M
\]

• If \(K_{\alpha} > \rho U^2 ec^2\pi\) then the spring and aerodynamic stiffness constitute a restoring force, which will balance the external moment.

• However, if \(K_{\alpha} < \rho U^2 ec^2\pi\) then the spring and aerodynamic stiffness do not constitute a restoring force. Instead of balancing the external moment, they add to it.

• It means that the static equilibrium is unstable.
Static Divergence (2)

- Static divergence in pitch occurs when the moment caused by the lift around the flexural axis is higher than the structural restoring force and of opposite sign.
- For every pitch stiffness there is an airspeed above which static divergence will occur.
- If this airspeed is outside the flight envelope of the aircraft, this is not a problem.
- The pitch-plunge model does not allow for static divergence in plunge.
- Again, this is because it ignores 3D effects.
Static Divergence (3)

- Remember that $e = \frac{x_f}{c} - \frac{1}{4}$.
- If the flexural axis lies on the quarter-chord (aerodynamic centre), then $e = 0$.
- Consequently, the moment of the lift around the flexural axis also becomes zero.
- It follows that static divergence is no longer possible.
- If $x_f$ is ahead of the aerodynamic centre, $e < 0$ and the static equilibrium equation becomes

$$\left(K_\alpha + \rho U^2 |e| c^2 \pi\right) \alpha = M$$

- Static divergence is no longer possible
Windmills

Pili windmill in Kos (1800)  Upminster Windmill (1803)  Bircham Windmill (1846)

The position of the flexural axis is very important for windmills