Aeroelasticity

Lecture 7:
Flight Flutter Testing

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Main reference

• A Historical Overview of Flight Flutter Testing, Michael W. Kehoe, NASA Technical Memorandum 4720, October 1995

• A lot of the stuff in this lecture is taken from this reference.
Flight flutter testing

• Despite all the efforts in developing design flutter tools, the only definitive method for clearing aircraft for flutter is flight testing

• All airworthiness and aircraft certification procedures require that aerospace constructors demonstrate that the flight envelope of a new aircraft is clear of flutter.

• In fact, for added security, there must be no flutter at 20% outside the flight envelope (15% for military aircraft)
Flight flutter history

• The first flight flutter tests were very basic:
  – Aircraft would be flown to all the extremes of their flight envelope.
  – If they survived then the aircraft was deemed safe
  – If they were destroyed then they had to be redesigned

• Clearly, this was not a satisfactory way of carrying out such tests.

• Von Schlippe performed the first formal flutter tests in 1935 in Germany
Von Schlippe’s test

- Von Schlippe flew the aircraft at an initially low airspeed.
- He vibrated the aircraft structures at its natural frequencies at each airspeed and plotted the resulting vibration amplitude.
- He predicted flutter when the amplitude reaches a high value (theoretically infinite).
- He estimated the natural frequencies of the structure during ground vibration tests.
Further history

• Von Schlippe’s technique continued to be used until a Junkers Ju90 aircraft fluttered in flight and crashed

• The problems with the procedure were:
  – Inadequate structural excitation in flight
  – Inaccurate measurement of response amplitude

• These problems could only be solved with better instrumentation and excitation capabilities - the method itself was sound
1940s

- The Americans used the same technique in the 1940s
- Example: Cessna AT-8 aircraft
Progress

• Von Schlippe’s flight flutter testing method was good but the instrumentation not very advanced.
• Between the 50s and 70s several advances in actuation and instrumentation brought about significant improvements in flight flutter testing.
• The response amplitude was replaced by the damping ratio as the flutter parameter.
F111 Flight test apparatus

Excitation using aerodynamic wing tabs

Diagram:
- Onboard recorder and TM transmitter
- Ground control and data processing
  - TM receiving and tape recording
  - Band pass filters
  - X-Y frequency sweep plotters
  - 18 channels accel and aero tabs
  - 8 channels flight parameters and control system (unfiltered)
- Test director
- Brush record of all transducers
- Manual data reduction (g versus Mach)
- Airplane tape
More modern apparatus
Excitation systems

• An ideal excitation system must:
  – Provide adequate excitation levels at all the frequency ranges of interest
  – Be light so as not to affect the modal characteristics of the structure
  – Have electrical or hydraulic power requirements that the aircraft can meet
Control surface pulses

• This method consists of impulsively moving one of the control surfaces and then bringing it back to zero.
• Theoretically, it is supposed to be a perfect impulse. Such an impulse will excite all the structure’s modes.
• In practice it is not at all perfect and can only excite modes of up to 10Hz.
• The transient response of the aircraft is easy to analyse for stability.
• However, high damping rates and lots of measurement noise can make this analysis difficult.
• The repeatability of pulses is low.
Oscillating control surfaces

• Instead of just pulsing the control surfaces, we oscillate them sinusoidally

• Three modes:
  – Dwell: Oscillation at constant frequency and amplitude
  – Frequency sweep: oscillation at constant amplitude but linearly increasing frequency
  – Amplitude sweep: oscillation at constant frequency but linearly increasing amplitude

• The demand signal is provided by the automatic control system. The excitation is accurate and can range from 0.1Hz to 100Hz.
Control surface excitation

FCC

‘FBI’ Command Digital Signal

Actuator

Control Surface
Control surface excitation (2)

- C-5 Galaxy – Flight flutter test using control surface excitation
- Airbus A350 – Flight flutter test using control surface excitation
Thrusters

• Aka bonkers, ballistic exciters, impulse generators
• These are small one-shot rockets that burn for 20ms and provide thrust up to 2,000Kg.
• They are attached at points that allow the measurement of particular modes of interest
• They are not used very much now. They have several disadvantages:
  – Single shot
  – Difficult to fire two or more simultaneously
  – Need thrusters of different burn times to excite different frequencies
Inertial exciters

Rotating eccentric weight or oscillating weight inertial exciters.

Many designs have been used.

They are not very popular nowadays.

Their excitation capability is low at low frequencies and too high at high frequencies.
Aerodynamic vanes

- Small winglets usually mounted on tip of a wing or a stabilizer
- The vanes are mounted on a shaft and oscillate around a mean angle
- The force depends on the size of the vane, the dynamic pressure and the oscillation angle
- They excite low frequencies adequately
- High frequency excitation depends on the frequency response of the mechanism
- Force depends on the square of the airspeed - at low speeds it is low
Aerodynamic vanes (2)

- Slotted rotating cylinder vane
- The cylinder oscillates, deflecting the airstream either upwards or downwards and creating an oscillating lift force
Random atmospheric turbulence

- This method is completely free and does not change the modal or control characteristics of the aircraft at all.
- On the other hand excitation levels can be low (we cannot ensure adequate levels of turbulence on test days)
- The signal-to-noise ratio of the response data is usually small
Von Karman Spectrum

• The frequency content of atmospheric turbulence is usually modelled using the Von Karman spectrum

\[
\Phi_{11}(\omega) = \sigma_g^2 \frac{L}{\pi} \frac{1}{\left(1 + \left(1.339 \frac{L}{V}\right)^2\right)^{5/6}} \quad \text{Longitudinal turbulence}
\]

\[
\Phi_{22}(\omega) = \sigma_g^2 \frac{L}{\pi} \frac{8}{3} \left(1 + \left(1.339 \frac{L}{V}\right)^2\right)^{11/6} \quad \text{Lateral turbulence}
\]

• Where \( \omega \) is the angular frequency, \( L=762 \text{m} \) is the length scale of atmospheric turbulence, \( V \) is the aircraft’s airspeed, \( \sigma_g=2.1-6.4 \) is the turbulence intensity
Von Karman example

Von Karman spectrum at an airspeed of 200m/s and $\sigma_g=2.1$.

It can be seen that most of the power is concentrated at very low frequencies, less than 1Hz.

The power at frequencies of 10Hz or more is very low
Comparison of two excitation systems

Response amplitude power spectra from exciter sweep and random turbulence

(a) Exciter sweep.

(b) Random turbulence.
# Summary of exciters

<table>
<thead>
<tr>
<th>Airplane</th>
<th>Surface</th>
<th>Location</th>
<th>Frequency range</th>
<th>Time to sweep, sec</th>
<th>Sweep law</th>
</tr>
</thead>
<tbody>
<tr>
<td>747</td>
<td>Wings</td>
<td>External vanes at wingtips</td>
<td>1.5–7.0 Hz</td>
<td>90</td>
<td>Exponential</td>
</tr>
<tr>
<td>DC-10</td>
<td>Wings horizontal</td>
<td>External vanes at tips of main surfaces</td>
<td>1–20 Hz and 1–10 Hz</td>
<td>90</td>
<td>Exponential</td>
</tr>
<tr>
<td>L-1011</td>
<td>Vertical tail</td>
<td>External vanes</td>
<td>1–18 Hz and 3–25 Hz</td>
<td>90</td>
<td>Linear period</td>
</tr>
<tr>
<td>S-3A</td>
<td>Wing stabilizer</td>
<td>External vanes</td>
<td>1.5–18 Hz and 3–25 Hz</td>
<td>90</td>
<td>Linear period</td>
</tr>
<tr>
<td>C-5A</td>
<td>Side of fuselage under stabilizer</td>
<td>External vanes on top of surfaces near tips</td>
<td>.5–25 Hz</td>
<td>60 normal 30 dive only</td>
<td>Exponential</td>
</tr>
<tr>
<td>F-14</td>
<td>Wing fin</td>
<td>Aero-tab External vane</td>
<td>5–50 Hz</td>
<td>15</td>
<td>Exponential</td>
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<tr>
<td>F-15</td>
<td>Normal control Ailerons Stabilator</td>
<td>External vane</td>
<td>2–16 Hz and 5–10 Hz</td>
<td>100–200</td>
<td>Linear frequency</td>
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<td>F-111</td>
<td>Wing</td>
<td>Aero-tab</td>
<td>35–2 Hz</td>
<td>45</td>
<td>Exponential</td>
</tr>
</tbody>
</table>
Excitation Signals

• There are four main types of excitation signals used:
  – Impulsive
  – Dwell
  – Sweep
  – Noise

• Dwell only excites one frequency at a time. Therefore, it is expensive since the test must last longer.

• Impulsive, sweep and noise excite many frequencies at a time
Frequency sweep (chirp)

Frequency sweep from 1Hz to 30Hz
Noise

Uniform noise from 1Hz to 30Hz

Time domain

Frequency domain
Real test data example

Data obtained during a flight flutter test.

Three dwells between 5Hz and 6Hz and one sweep from 5Hz to 7Hz.

Excitation is control surface deflection.
Data Analysis

• Once the excitation has been applied, the aircraft structure’s response is measured at several locations (e.g. wingtip, tail tip, engine mounts etc) using accelerometers.
• The response data and the excitation data are saved and transferred to a ground station for analysis
• The analysis uses simple but effective modal analysis tools
Modal Analysis (1)

• As only one excitation is applied at any one time, the system is Single Input Multiple Output (SIMO)

• Denote by $y_i(t)$ the $i$th measured response and by $f(t)$ the excitation force

• The $i$th Frequency Response Function of the system is defined as:

$$H_i(\omega) = \frac{Y_i(\omega)}{F(\omega)}$$

• Where $Y_i(\omega)$ is the Fourier Transform of the $y_i(t)$ signal and $F(\omega)$ is the Fourier Transform of the $f(t)$ signal.
• Notice that better FRF estimators could be applied but are not used in practice. The emphasis is on speed and simplicity.
• The FRFs are plotted and inspected by the test operator, along with the time domain responses and the response predictions from an aeroelastic mathematical model.
• The FRFs are also analysed in order to extract the natural frequencies and damping ratios.
Simulated Example

Excitation force and three responses from a simulated flight flutter tests.

The excitation signal is a chirp with frequencies from 1Hz to 45Hz.

The natural frequencies of the aircraft on the ground are 8Hz, 16Hz and 39Hz.
All three FRFs show that there are three modes in the interval from 1Hz to 45Hz.

Only two modes are clearly visible in response 1. Response 3 is best for observing all three modes.

This demonstrates that it is important to analyse many responses from the aircraft.
Effect of airspeed

The airspeed affects all three modes. The height of the peaks changes with airspeed.

The higher the peak, the lower the damping.

The 2nd mode is of particular interest. First the height falls, then it increases and at V=40m/s it is very high. This is the mode whose damping will go to zero at flutter.
Parameter Estimation

- The damping ratio and natural frequency of each mode are the parameters of the mode.
- They must be estimated in order to determine how close the system is to flutter.
- There are many parameter estimation methods, ranging from the simple to the most accurate.
- The quality and resolution of data available from flight flutter tests suggests that simpler methods should be used.
- The simplest method is the Half Power Point.
The Half Power Point method is a graphical approach and, therefore, not very accurate.

Nevertheless, it is always used, even when more sophisticated parameter estimation techniques are applied.

The best algorithms and computers are no replacement for an engineer with a ruler and plotting paper, apparently.
Rational Fraction Polynomials (1)

• The FRF of any dynamic system can be written as:

\[ H(\omega) = \frac{b_{nb}(i\omega)^{nb} + b_{nb-1}(i\omega)^{nb-1} + \cdots + b_0}{(i\omega)^{na} + a_{na-1}(i\omega)^{na-1} + \cdots + a_0} \]

• Where the coefficients \( a_i, b_i \) are to be estimated from \( L \) measured values of the FRF at \( L \) frequency values.

\[
\begin{bmatrix}
H(\omega_1)(i\omega_1)^{na} \\
\vdots \\
H(\omega_L)(i\omega_L)^{na}
\end{bmatrix} = -
\begin{bmatrix}
H(\omega_1)(i\omega_1)^{na-1} & \cdots & H(\omega_1)(i\omega_1)^{nb} & \cdots & 1 \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
H(\omega_L)(i\omega_L)^{na-1} & \cdots & H(\omega_L)(i\omega_L)^{nb} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
a_{na-1} \\
\vdots \\
a_0 \\
b_{nb} \\
b_0
\end{bmatrix}
\]
Rational Fraction Polynomials (2)

- The denominator is the system’s characteristic polynomial.
- Once the $a_i, b_i$ coefficients are estimated, the system eigenvalues can be calculated from the roots of the denominator.
- The polynomial orders $na$ and $nb$ are usually given by $na=2m$ and $nb=2m-1$ where $m$ is the number of modes that we desire to model.
- In order to allow for experimental and signal processing errors, the polynomial order can be chosen to be higher than $2m$. 
Latest modal analysis

• Until recently, only very basic modal analysis was used in flight flutter testing

• The quality of data, the number of transducers and the cost of the flight testing programme prohibited the use of more sophisticated methods.

• These days, more and more high end modal analysis is introduced in flight flutter testing, e.g.
  – Polyreference methods
  – Stabilization diagrams
  – Operational modal analysis
  – Model updating
Damping trends

• The damping ratio trends are plotted and a linear extrapolation is usually performed to determine whether the next planned flight condition will be tested.

• This is the most important part of the flight flutter test. The point of the test is not to reach the flutter point, nor to predict it accurately. It is to clear the flight envelope.

• If the flight envelope has been cleared (i.e. all flight points tested) the test is finished

• If a flight point is deemed unsafe (i.e. too close to flutter), the test is finished.
Modal parameter variation

This is the complete variation of the modal parameters with airspeed.

However, a test director will obtain these results gradually, one result at each test point.
Initially, the damping increases. Therefore the flight conditions are considered safe until $V=30\text{m/s}$. At $V=40\text{m/s}$ the damping ratio of mode 2 drops suddenly and significantly.

The flight condition is near critical and the flight flutter test is terminated.
Damping Extrapolation

• An estimate of the stability of each flight condition can be obtained if the damping ratio is plotted against dynamic pressure. The resulting graphs are nearly linear.

• At each flight condition the last two measured damping ratio values can be linearly extrapolated to estimate the flutter flight condition.

• If $\mathbf{d}$ is the vector containing the damping ratio measurements for mode 2 and $\mathbf{q}$ the vector containing the flight dynamic pressures:

$$q_{crit} = -c/a \text{ where } \mathbf{d} = \begin{bmatrix} q_1 & 1 \end{bmatrix} \begin{bmatrix} a \cr c \end{bmatrix}$$
At $V=35\text{m/s}$ the predicted flutter speed is over $70\text{m/s}$. 

At $V=40\text{m/s}$ the predicted flutter speed is $48\text{m/s}$. 

The true flutter speed is $44\text{m/s}$.
Hard flutter

- Hard flutter is characterized by a very sudden drop in damping ratio:
Other stability criteria

• It is clear that the damping ratio can be misinterpreted as a stability criterion.

• Alternative stability criteria have been proposed and some of them are used in practice.

• The most popular of these are:
  – The Flutter Margin
  – The envelope function
Flutter Margin

• The Flutter Margin is defined for the case of a classical binary flutter mechanism.
• The aircraft may have many modes but the Flutter Margin procedure is only applied to the two modes that combine to cause flutter.
• The characteristic polynomial is of the form:
  \[ a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 \]
• And the Routh stability criterion requires that:
  \[ a_3 a_2 a_1 - a_4 a_1^2 - a_0 a_3^2 = 0 \]
Flutter Margin (2)

- Without loss of generality we can see $a_4 = 1$ and divide by $a_3^2$ to get:

$$F = -\left(\frac{a_1}{a_3}\right)^2 + a_2\left(\frac{a_1}{a_3}\right) - a_0 = 0$$

- where $F$ is called the Flutter Margin. Writing the four eigenvalues as

$$\lambda_1 = \beta_1 + i\omega_1, \lambda_2 = \beta_1 - i\omega_1, \lambda_3 = \beta_2 + i\omega_2, \lambda_4 = \beta_2 - i\omega_2$$

- yields

$$F = \left[\left(\frac{\omega_2^2 - \omega_1^2}{2}\right) + \left(\frac{\beta_2^2 - \beta_1^2}{2}\right)\right]^2 + 4\beta_1\beta_2\left[\left(\frac{\omega_2^2 + \omega_1^2}{2}\right) + 2\left(\frac{\beta_2^2 + \beta_1^2}{2}\right)\right]^2 - \left[\left(\frac{\beta_2 - \beta_1}{\beta_2 + \beta_1}\right)\left(\frac{\omega_2^2 - \omega_1^2}{2}\right) + \left(\frac{\beta_2^2 + \beta_1^2}{2}\right)^2\right]$$
Flutter Margin (3)

Therefore, by measuring the natural frequencies and damping ratios of the two modes at each airspeed we can calculate the flutter margin since:

\[ \beta_1 = \omega_{n,1} \zeta_1, \quad \omega_1 = \omega_{n,1} \sqrt{1 - \zeta_1^2}, \quad \beta_2 = \omega_{n,2} \zeta_2, \quad \omega_2 = \omega_{n,2} \sqrt{1 - \zeta_2^2}, \]

- If \( F > 0 \) then the aircraft is aeroelastically stable. If \( F \) begins to approach 0, then the aircraft is near flutter.
Variation of $F$ with flight condition

- Using the pitch-plunge quasi-steady equations, it can be shown that the ratio $a_1/a_3$ is proportional to the dynamic pressure, i.e.

$$\frac{a_1}{a_3} \propto q, \quad q = \frac{1}{2} \rho U^2$$

- Therefore, the Flutter Margin is a quadratic function of dynamic pressure

$$F = B_2 q^2 + B_1 q + B_0$$
Flutter Margin conclusions

• So the Flutter Margin is as good a stability criterion as the damping ratio.
• Additionally, its variation with airspeed and density is known.
• Well, not really. All true aeroelastic systems are unsteady, not quasisteady.
• Therefore, $F$ is not really a known function of $q$. On the other hand, $F$ behaves more smoothly than the damping ratio in the case of hard flutter.
Comparison to damping ratio

FM drop near flutter is still abrupt for a hard flutter case.

However, it is less abrupt than the drop of the damping ratio...
Envelope Function

• The envelope function is the absolute value of the analytic signal.
• It defines the envelope in which the function oscillates.
• The analytic signal of a function $y(t)$ is given by

$$Y(t) = y(t) + iy_h(t)$$

• Where $y_h(t)$ is the Hilbert Transform of $y(t)$
The Hilbert Transform of $y(t)$ is defined as

$$y_h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{y(\tau)}{t-\tau} d\tau$$

So it is a convolution of the function over all times.

It can be more easily calculated from the Fourier Transform of $y(t)$, $Y(\omega)$

$$Y_h(\omega) = -j \frac{\omega}{|\omega|} Y(\omega)$$

where $\omega$ is the frequency in rad/s
Hilbert Transform (2)

- Transforming back into the time domain and noting that only positive frequencies are of interest gives

\[ y_h(t) = F^{-1} \left( \text{Im}(Y(\omega)) - j \text{Re}(Y(\omega)) \right) \]

- Where \( F^{-1} \) is the inverse Fourier Transform.

- Then the envelope function is calculated from

\[ E(t) = |Y(t)| = \sqrt{y^2(t) - y_h^2(t)} \]

- However, the easiest way of calculating the envelope function is to use Matlab’s \texttt{hilbert} function.
Example of envelope
Envelope variation with flight condition
Time centroid

• With the envelope function method, the stability criterion is the position of the time centroid of the envelope.

• The time centroid is given by

\[ \bar{t} = \frac{\int_0^{t_1} E(t) t \, dt}{\int_0^{t_1} E(t) \, dt} \]

• Where \( t_1 \) is a reference time representing the duration of the response signals.
At flutter, the time centroid is close to the centre of the time window, i.e. $t_1/2$.

The stability criterion is then

$$ S = \frac{1}{t} - \frac{2}{t_1} $$

And it is close to $S=0$ at the flutter condition.
Variation of $S$ with flight condition

Example of wind tunnel flutter test with envelope function-based stability criterion
Conclusion

• Flight flutter testing is still as much an art as it is a science
• The best flutter predictions are obtained when the aircraft is flown near the flutter flight condition
• If this condition is inside the flight envelope the test can be very dangerous
• Good excitation, good instrumentation, good data analysis and a lot of experience are needed for a successful flight flutter test