Flight Dynamics and Control

Lecture 4: Lateral stability Derivatives

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Previously on AERO0003-1

• We developed linearized equations of motion

Longitudinal direction

\[
\begin{bmatrix}
m & -\ddot{X}_w & 0 & 0 \\
0 & (m - \ddot{Z}_w) & 0 & 0 \\
0 & -\dddot{M}_w & I_y & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\nu} \\
\dot{\nu} \\
\dot{\dot{q}} \\
\dot{\dot{\theta}} \\
\end{bmatrix}
+ \begin{bmatrix}
-\dddot{X}_u & -\dddot{X}_w & -(\dddot{X}_q - mW_e) & mg \cos \theta_e \\
-\dddot{Z}_u & -\dddot{Z}_w & -(\dddot{Z}_q + mU_e) & mg \sin \theta_e \\
-\dddot{M}_u & -\dddot{M}_w & -\dddot{M}_q & 0 \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\nu \\
w \\
\dot{q} \\
\dot{\theta} \\
\end{bmatrix}
= \begin{bmatrix}
\dddot{X}_\eta & \dddot{X}_\tau \\
\dddot{Z}_\eta & \dddot{Z}_\tau \\
\dddot{M}_\eta & \dddot{M}_\tau \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\eta \\
\tau \\
\end{bmatrix}
\]

Lateral direction

\[
\begin{bmatrix}
m & 0 & 0 & 0 & 0 \\
0 & I_x & -I_{xz} & 0 & 0 \\
0 & -I_{xz} & I_z & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\nu} \\
\dot{\dot{p}} \\
\dot{\dot{r}} \\
\dot{\phi} \\
\dot{\psi} \\
\end{bmatrix}
+ \begin{bmatrix}
-\dddot{Y}_v & -(\dddot{Y}_p + mW_e) & -(\dddot{Y}_r - mU_e) & -mg \cos \theta_e & -mg \sin \theta_e \\
-\dddot{L}_v & -\dddot{L}_p & -\dddot{L}_r & 0 & 0 \\
-\dddot{N}_v & -\dddot{N}_p & -\dddot{N}_r & 0 & 0 \\
0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\nu \\
p \\
r \\
\phi \\
\psi \\
\end{bmatrix}
= \begin{bmatrix}
\dddot{Y}_\xi & \dddot{Y}_\zeta \\
\dddot{L}_\xi & \dddot{L}_\zeta \\
\dddot{N}_\xi & \dddot{N}_\zeta \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\zeta \\
\phi \\
\psi \\
\psi \\
\end{bmatrix}
\]
Lateral stability derivatives

- We have already derived expressions for the most important longitudinal stability derivatives.
- Here we will do the same thing for the lateral stability derivatives.
- This means the derivatives of the sideforce $Y$, rolling moment $L$ and yawing moment $N$ with respect to perturbations $\nu$, $p$, $r$, $\xi$, $\zeta$.
- The discussion is based on
  - Etkin and Reid, Dynamics of flight.
  - Roskam, Methods for estimating stability and control derivatives of conventional subsonic airplanes.
- Both of these works include results from:
  - Hoak et al, USAF Stability and Control Datcom (Data compendium)
Sideslip angle

- The angle of sideslip is a lateral angle of attack.
- It should not be confused with the yaw angle.

\[ \beta = \sin^{-1} \frac{V}{V_0} \]
Equilibrium flight condition

- Consider rectilinear flight at a constant sideslip angle $\beta_e$:
  \[ V = V_e + v, \quad \beta_e = \sin^{-1} \frac{V_e}{V_0} \]

- The perturbed sideslip angle is given by:
  \[ \beta = \sin^{-1} \frac{V_e + v}{V_0} \approx \sin^{-1} \frac{V_e}{V_0} + \frac{v}{V_0} = \beta_e + \frac{v}{V_0} \]

- Take the difference form:
  \[ d\beta = d\left( \beta_e + \frac{v}{V_0} \right) = \frac{1}{V_0} dv \]

- This means that derivatives with respect to $\beta$ are in fact derivatives with respect to $v$. 
Sideforce

• The wing, fuselage, propeller and fin all contribute to the sideforce $Y$.

• Wing sideforce contribution:
  – Sidewash factor
  – Wing sideforce

• Fuselage sideforce contribution:
  – Sidewash factor
  – Fuselage lift

• Propeller sideforce contribution:
  – Sidewash factor
  – Propeller normal force

• Fin sideforce contribution:
  – Fin lift
Rolling moment

• The wing, fin and fuselage contribute to the rolling moment.

• Wing rolling moment contribution:
  – Wingtip vortex effect
  – Dihedral effect
  – Sweep effect

• Fuselage rolling moment contribution:
  – High wing or low wing

• Fin rolling moment contribution:
  – Fin lift
Yawing moment

- The wing, fin and fuselage contribute to the yawing moment.
- Wing yawing moment contribution:
  - Sidewash factor
  - Angle of attack difference on two half-wings due to rolling:
    - Asymmetric induced drag
  - Angle of attack difference on two half-wings due to yawing:
    - Asymmetric lift and drag, particularly when the wing is swept
- Fuselage yawing moment contribution:
  - Sidewash factor
  - Fuselage lift
- Fin yawing moment contribution:
  - Fin lift
- Propeller yawing moment contribution:
  - Propeller normal force
Fin lift

- A fin is an upright wing. Depending on the geometry, it can be a half-wing.
- In general, the fin will have a symmetric airfoil and its geometric angle of attack will be zero.
- The fin lift is then

\[ C_{LF} = c_1 \alpha_F + c_2 \zeta \]

- Where \( c_1 \) is the fin’s lift curve slope, \( c_2 \) is the rudder curve slope, \( \alpha_F \) is the fin angle of attack and \( \zeta \) is the rudder deflection angle.
Lift curve slope

• The lift curve slope for a wing is given by:
  – Straight tapered wings:

\[ C_{L\alpha} = 0.995 \frac{c_{l\alpha}}{E + c_{l\alpha} / \pi AR}, \text{ where } E = 1 + \frac{2\lambda}{AR(1 + \lambda)} \]

  – Swept wings:

\[ \beta C_{L\alpha} = \frac{2\pi}{\sqrt{\frac{2}{\beta AR} + \left( \frac{1}{k^2 \cos^2 \Lambda_{\beta}} + \left( \frac{2}{\beta AR} \right)^2 \right)^2}} \]

\[ \beta = \sqrt{1 - M^2} \]

where \( \tan \Lambda_{\beta} = \frac{\tan \Lambda_{1/2}}{\beta} \) and \( k = \frac{\beta c_{l\alpha}}{2\pi} \)

• For half-wing fins, we can divide \( \alpha_F \) by two but the aspect ratio must be for a full wing.
Fin angle of attack

• The angle of attack of the fin depends on:
  – Sideslip angle $\beta$
  – Sidewash angle $\sigma$
  – Roll velocity $p$
  – Yaw velocity $r$

• The last two motions introduce effective curvature and camber to the fin but these effects can be modelled as angle of attack modifications.
Sidewash angle

- The wing and fuselage deflect the flow seen by the tail.
- The deflection angle is known as the sidewash angle $\sigma$.
- The angle of attack of the fin is:

$$\alpha_F = -\beta + \sigma$$

- We assume that the sidewash is a linear function of sideslip, so that:

$$\alpha_F = -\beta + \frac{\partial \sigma}{\partial \beta} \beta = -\left(1 - \frac{\partial \sigma}{\partial \beta}\right) \beta$$
Sidewash derivative approximation

- The sidewash derivative with respect to $\beta$ can be approximated as:

$$\frac{\partial \sigma}{\partial \beta} = -0.276 + 3.06 \frac{S_F}{S} \frac{1}{1 + \cos \Lambda_{c/4}} + 0.4 \frac{Z_w}{d} + 0.009 A$$

- where $Z_w$ is the vertical distance from the wing root quarter-chord to the fuselage centreline, positive downward, and

$$d = \sqrt{\frac{\text{average fuselage cross sectional area}}{0.7854}}$$

- $S_F$ is the fin area, $S$ is the wing area, $A$ the aspect ratio and $\Lambda_{c/4}$ the sweep angle at the quarter-chord.
Angle due to roll velocity

• The roll velocity introduces a linearly varying velocity distribution on the fin, $p_z$.

• A mean value for this velocity distribution is $p_z F$, where $z_F$ is an appropriate mean height of the fin.

• We can simplify by saying that $z_F$ is the height of the fin’s aerodynamic centre.

• Note that the roll motion also has an effect on the sidewash angle.

• Then the change in angle of attack due to the roll motion is

$$\Delta \alpha_p = -\frac{p z_F}{V_0} + \frac{\partial \sigma}{\partial p} p$$
Angle due to yaw velocity

• The yaw velocity $r$ introduces an additional airspeed $rl_F$ perpendicular to the fin.

• Yawing also has an effect on the sidewash angle.

• Then, the angle of attack change due to the yaw velocity is:

$$\Delta \alpha_r = \frac{rl_F}{V_0} + \frac{\partial \sigma}{\partial r} r$$
Sideforce due to fin lift

• The total sideforce due to fin lift is then given by:

\[ Y_a = \frac{1}{2} \rho V_0^2 S_F c_1 \left( -\left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta - \frac{p_{z_F}}{V_0} \frac{\partial \sigma}{\partial p} + \frac{r l_F}{V_0} + \frac{\partial \sigma}{\partial r} \right) + \frac{1}{2} \rho V_0^2 S_F c_2 \xi \]

• Substituting \( \beta = \beta_e + v/V_0 \):

\[ Y_a = \frac{1}{2} \rho V_0^2 S_F c_1 \left( -\left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta_e - \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \frac{v}{V_0} - \frac{p_{z_F}}{V_0} \frac{\partial \sigma}{\partial p} + \frac{r l_F}{V_0} + \frac{\partial \sigma}{\partial r} \right) + \frac{1}{2} \rho V_0^2 S_F c_2 \xi \]

• Separating the stiffness from the damping terms we get:

\[ Y_a = Y_{a_e} + \frac{1}{2} \rho V_0^2 S_F c_1 \left( \frac{\partial \sigma}{\partial p} p + \frac{\partial \sigma}{\partial r} r \right) + \frac{1}{2} \rho V_0^2 S_F c_1 \left( -\left(1 - \frac{\partial \sigma}{\partial \beta} \right) v + r l_F - p_{z_F} \right) + \frac{1}{2} \rho V_0^2 S_F c_2 \xi \]
The aerodynamic derivatives of $Y$ due to fin

- The aerodynamic derivatives of $Y$ become:

\[
\begin{align*}
\tilde{Y}_v &= -\frac{1}{2} \rho V_0 S_F c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \\
\tilde{Y}_p &= \frac{1}{2} \rho V_0^2 S_F c_1 \frac{\partial \sigma}{\partial p} - \frac{1}{2} \rho V_0 S_F c_1 z_F \\
\tilde{Y}_r &= \frac{1}{2} \rho V_0^2 S_F c_1 \frac{\partial \sigma}{\partial r} + \frac{1}{2} \rho V_0 S_F c_1 l_F \\
\tilde{Y}_\zeta &= \frac{1}{2} \rho V_0^2 S_F c_2
\end{align*}
\]
DATCOM values for sideforce derivatives due to fin

- The terms $\partial \sigma / \partial p$ and $\partial \sigma / \partial r$ are difficult to calculate.
- The DATCOM proposes the following value for the sideforce derivative due to roll:
  $$Y_p = 2Y_v \frac{z_F \cos \alpha - l_F \sin \alpha}{b} = -2 \frac{S_F}{S} c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \frac{z_F \cos \alpha_e - l_F \sin \alpha_e}{b}$$
  - where $\alpha_e$ is the equilibrium angle of attack.
- Similarly, for the sideforce derivative due to yaw, the DATCOM proposes
  $$Y_r = -2Y_v \frac{z_F \sin \alpha + l_F \cos \alpha}{b} = 2 \frac{S_F}{S} c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \frac{z_F \cos \alpha_e - l_F \sin \alpha_e}{b}$$
  - Note that both of these derivatives are only the contributions from the fin. Contributions from the body and wing will be given later.
  - Furthermore, these contributions are generally small.
Rolling moment diagram

• The fin generates a rolling moment around the centre of gravity.

• The rolling moment is equal to the fin lift times the distance between the fin’s aerodynamic centre and the centre of gravity.
Rolling moment due to fin

- The rolling moment due to the fin is given simply by:

\[ L_a = \frac{1}{2} \rho V_0^2 S_F C_{LF} z_F = \frac{1}{2} \rho V_0^2 S_F (c_1 \alpha_F + c_2 \xi) z_F \]

- Substituting for the fin angle of attack:

\[ L_a = \frac{1}{2} \rho V_0^2 S_F \left( c_1 \left( -\left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta - \frac{pz_F}{V_0} + \frac{\partial \sigma}{\partial p} p + \frac{rl_F}{V_0} + \frac{\partial \sigma}{\partial r} r \right) + c_2 \xi \right) z_F \]

- Substituting \( \beta = \beta_e + v/V_0 \):

\[ L_a = L_{a_e} + \frac{1}{2} \rho V_0^2 S_F z_F c_1 \left( \frac{\partial \sigma}{\partial p} p + \frac{\partial \sigma}{\partial r} r \right) + \frac{1}{2} \rho V_0 S_F z_F c_1 \left( -\left(1 - \frac{\partial \sigma}{\partial \beta} \right) v - pz_F + rl_F \right) + \frac{1}{2} \rho V_0^2 S_F z_F c_2 \xi \]
The aerodynamic derivatives of $L$ become (recalling that $V_F = \frac{S_F l_F}{S_b}$):

\[
\begin{align*}
\tilde{L}_v &= -\frac{1}{2} \rho V_0 S_F z_F c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \\
\tilde{L}_p &= \frac{1}{2} \rho V_0^2 S_F z_F c_1 \frac{\partial \sigma}{\partial p} - \frac{1}{2} \rho V_0 S_F c_1 z_F^2 \\
\tilde{L}_r &= \frac{1}{2} \rho V_0^2 S_F z_F c_1 \frac{\partial \sigma}{\partial r} + \frac{1}{2} \rho V_0 S_F z_F c_1 l_F \\
\tilde{L}_\zeta &= \frac{1}{2} \rho V_0^2 S_F z_F c_2
\end{align*}
\]

\[
\begin{align*}
L_v &= -V_F \frac{z_F}{l_F} c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \\
L_p &= V_F c_1 \frac{z_F}{l_F} \left( V_0 \frac{\partial \sigma}{\partial p} - z_F \right) \\
L_r &= V_F \frac{z_F}{l_F} c_1 \left( V_0 \frac{\partial \sigma}{\partial r} + l_F \right) \\
L_\zeta &= V_F \frac{z_F}{l_F} c_2
\end{align*}
\]
DATCOM values for rolling moment derivatives due to fin

- Again, the terms $\partial \sigma / \partial \rho$ and $\partial \sigma / \partial r$ are difficult to calculate.
- The DATCOM proposes the following value for the rolling moment due to roll:
  \[ L_p = 2 Y_v \frac{z_F^2}{b} = -2 \frac{S_F}{S} c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \frac{z_F^2}{b} \]

- Similarly, for the rolling moment derivative due to yaw, the DATCOM proposes
  \[ L_r = 2 \frac{S_F}{S} c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \left( z_F \sin \alpha + l_F \cos \alpha \right) \left( z_F \cos \alpha_e - l_F \sin \alpha_e \right) \frac{b^2}{b^2} \]

- Note that both of these derivatives are only the contributions from the fin. Contributions from the body and wing will be given later.
- Furthermore, these contributions are generally small.
The fin lift causes a restoring yawing moment around the aircraft’s CG.

Assuming that the sideslip angle is small, the moment is given by:

\[ N_a = L_F l_F \]
**Yawing moment due to fin**

- The yawing moment due to the fin is given simply by:

\[ N_a = -\frac{1}{2} \rho V_0^2 S_F C_{LF} l_F = -\frac{1}{2} \rho V_0^2 S_F (c_1 \alpha_F + c_2 \zeta) l_F \]

- Substituting for the fin angle of attack:

\[ N_a = -\frac{1}{2} \rho V_0^2 S_F \left( c_1 \left( -\left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \beta - \frac{p z_F}{V_0} + \frac{\partial \sigma}{\partial p} p + \frac{r l_F}{V_0} + \frac{\partial \sigma}{\partial r} r \right) + c_2 \zeta \right) l_F \]

- Substituting \( \beta = \beta_e + v/V_0 \):

\[ N_a = N_{ae} - \frac{1}{2} \rho V_0^2 S_F l_F c_1 \left( \frac{\partial \sigma}{\partial p} p + \frac{\partial \sigma}{\partial r} r \right) + \frac{1}{2} \rho V_0 S_F l_F c_1 \left( \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) v + p z_F - r l_F \right) - \frac{1}{2} \rho V_0^2 S_F l_F c_2 \zeta \]
$N$ aerodynamic derivatives due to fin

- The aerodynamic derivatives of $N$ become (recalling that $V_F = \frac{S_F l_F}{S_b}$):

  \[ \tilde{N}_v = \frac{1}{2} \rho V_0 S_F l_F c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \]

  \[ N_v = \frac{S_F l_F}{S_b} c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) = V_F c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \]

  \[ \tilde{N}_p = -\frac{1}{2} \rho V_0^2 S_F l_F c_1 \frac{\partial \sigma}{\partial p} + \frac{1}{2} \rho V_0 S_F l_F c_1 z_F \]

  \[ N_p = V_F c_1 \left( z_F - V_0 \frac{\partial \sigma}{\partial p} \right) \]

  \[ \tilde{N}_r = -\frac{1}{2} \rho V_0^2 S_F l_F c_1 \frac{\partial \sigma}{\partial r} - \frac{1}{2} \rho V_0 S_F l_F^2 c_1 \]

  \[ N_r = -V_F c_1 \left( V_0 \frac{\partial \sigma}{\partial r} + l_F \right) \]

  \[ \tilde{N}_\xi = -\frac{1}{2} \rho V_0^2 S_F l_F c_2 \]

  \[ N_\xi = -V_F c_2 \xi \]
DATCOM values for yawing moment derivatives due to fin

- Again, the terms $\frac{\partial \sigma}{\partial \rho}$ and $\frac{\partial \sigma}{\partial r}$ are difficult to calculate.
- The DATCOM proposes the following value for the yawing moment due to roll:
  \[ N_p = L_r \]
- Similarly, for the yawing moment derivative due to yaw, the DATCOM proposes
  \[ N_r = -2 \frac{S_F}{S} c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) \frac{(z_F \sin \alpha + l_F \cos \alpha)^2}{b^2} \]
- Note that both of these derivatives are only the contributions from the fin. Contributions from the body and wing will be given later.
- Furthermore, these contributions are generally small.
Static stability in yaw

- Static stability in yaw is also known as weathercock or yaw stiffness.
- At static conditions, $p=r=0$ and $\beta$ is a constant.
- The yawing moment $N$ becomes:
  \[
  N_a = -\frac{1}{2} \rho V_0^2 S_F l_F \left(-c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \beta + c_2 \delta\right)
  \]
- Non-dimensionalise with respect to $\frac{1}{2} \rho V_0^2 Sb$
  \[
  C_N = -\frac{S_F l_F}{Sb} \left(-c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \beta + c_2 \delta\right) = -\bar{V}_F \left(-c_1 \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \beta + c_2 \delta\right)
  \]
Static stability in yaw (2)

- The aircraft is stable in yaw if $\frac{\partial C_N}{\partial \beta} < 0$

- where

$$\frac{\partial C_N}{\partial \beta} = V_F \left( c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) - c_2 \frac{\partial \delta}{\partial \beta} \right)$$

- If the rudder is held fixed, we obtain the controls fixed yaw stability criterion:

$$\frac{\partial C_N}{\partial \beta} = V_F c_1 \left( 1 - \frac{\partial \sigma}{\partial \beta} \right) < 0$$

- Recall that this is only the fin contribution to stability in yaw. The wing, body and propeller contributions must also be included in order to properly assess the stability of the aircraft.
Effect of sideslip on wing

- Straight rectangular wing with AR=8 at $\alpha=5^\circ$ and $\beta=10^\circ$.
- The left wingtip vortex moves away from the wing.
- The right wingtip vortex moves towards the wing.
- Sideslip has a significant effect on downwash and therefore induced drag.
- The effect on the lift is small.
- The sideforce is small.
Increasing angle $\beta$
Discussion of effect of $\beta$ on wings

- For planar wings, the main effect of sideslip is an asymmetry in drag.
  - The trailing wing has high drag and the leading wing has low drag.
  - This asymmetry causes a destabilising yawing moment.
- There is a small amount of asymmetry in lift very near the wingtips but it can be neglected.
- The sideforce is also very low and can be neglected.
- The main lateral load contribution of a planar wing in sideslip is then the yawing moment $N$. Even this moment can be small and neglected if there is no dihedral.
Sideslip and dihedral

- Adding dihedral changes completely the situation:
  - Same wing as before at $\beta = 10^\circ$ but with $10^\circ$ dihedral.
Dihedral effect

• In the presence of dihedral, the yawing moment caused by sideslip changes direction:
  – The trailing wing now has low drag and the leading wing high drag.
  – The yawing moment is stabilizing.

• On the other hand, the lift is now completely asymmetric.
  – The leading wing has high lift and the trailing wing low lift.
  – A rolling moment is created.

• Anhedral has exactly the opposite effect:
  – The yawing moment is destabilising.
  – The rolling moment is of opposite direction.
Sideslip and sweep

- Sweep also has an effect:
  - Same wing as before at $\beta=10^\circ$ but with $30^\circ$ sweep, no dihedral.
Sweep effect

• Sweep does not affect the direction of the yawing moment:
  – The trailing wing has high drag and the leading wing low drag.
  – The yawing moment is destabilizing but weak.

• It does however create a rolling moment, albeit weaker than that created by dihedral.
  – The leading wing has high lift and the trailing wing low lift.
  – A rolling moment is created.

• Forward swept wings do not produce a significant rolling moment.
Roll angle

- Aircraft at a roll angle are usually subjected to a restoring moment.
- The lift is inclined so the aircraft will start sideslipping.
- If the wing has dihedral, the sideslip will cause a stabilising rolling moment.
- The low wing is also the leading wing and it has high lift.
- This phenomenon is known as roll stiffness.
Roll angle and dihedral

- Dihedral has an effect when the wing is rolled, even if the sideslip angle is $0^\circ$:
  - Same wing as before at $\beta=0^\circ$ but with $15^\circ$ dihedral at a $10^\circ$ roll angle.
Dihedral effect on roll

• Dihedral causes mainly a lift asymmetry when the wing is rolled:
  – The high wing has low lift and the low wing high lift.
  – The result is a restoring rolling moment.
• It also creates a yawing moment.
  – The high wing has low drag and the low wing high drag.
• The effect of dihedral is increased when the aircraft starts sideslipping.
• Anhedral has exactly the opposite effect.
Effect of fuselage on roll moment

- The position of the wing on the fuselage has a significant effect on roll stability.
  - High wing: stabilising moment
  - Low wing: destabilising moment.
- This effect is due to the fact that the fuselage diverts the flow around the wing.
Roll Control

- Roll control can be accomplished using the ailerons

The total aileron angle is given by

$$\delta_a = (\delta_1 + \delta_2)/2$$
Aileron adverse yaw

- Increasing the lift also increases the drag and vice versa.
- When deflecting ailerons, there is a net yawing moment in an opposite direction to the rolling moment.
  - When rolling left (in order to turn left), there is a yawing moment to the right
  - This can make turning very difficult, especially for high aspect ratio wings
Roll control by spoilers

• Another way of performing roll control is by deforming a spoiler on the wing towards which we want to turn. To turn left:

The spoiler decreases lift and increases drag. Now the yawing moment is in the same direction as the roll.

‘Proverse yaw’
Frise Ailerons

The idea is to counteract the higher lift induced drag of the down wing with higher profile drag on the up wing. Frise ailerons are especially designed to create very high profile drag when deflected upwards. When deflected downwards the profile drag is kept low. Thus, they alleviate or, even, eliminate adverse yaw.
Differential Aileron Deflection

- The roll rate of the aircraft depends on the mean aileron deflection angle. The individual deflections $\delta_1$ and $\delta_2$ do not have to be equal.

Differential deflection means that the up aileron is deflected by a lot while the down aileron is deflected by a little.
Effect of roll rate

- Roll rate creates significant moments:
  - Same wing as before at $\alpha=0^\circ$ with 18$^\circ$/s roll rate (three times faster than a watch).
Roll rate effect

• The plunging wing has high lift while the climbing wing has low lift:
  – A significant rolling moment is generated, known as roll damping

• The plunging wing also has high drag while the climbing wing has low drag:
  – A significant yawing moment is generated
Roll subsidence

- The pilot deflects the ailerons in order to start rolling:
  - The climbing wing has high lift, the plunging wing low lift.
- As the aircraft accelerates in roll, the lift of the climbing wing decreases and that of the plunging wing increases.
- This means that the roll rate cannot accelerate to very high values:
  - As the roll rate increases, the rolling moment decreases and eventually falls to zero.
- This phenomenon is know as roll subsidence.
Wing and body contribution to $Y_v$

- The wing contribution to $Y_v$ (non-dimensional) is given by:

$$Y_v = -0.0001 |\Gamma| \frac{180}{\pi} \text{ per rad}$$

- where $\Gamma$ is the wing dihedral.

- The body contribution to $Y_v$ (non-dimensional) is given by:

$$Y_v = -2K_i \frac{S_0}{S} \text{ per rad}$$

- Where $S_0$ is the cross-sectional area of the fuselage at the point $x_0$ along the body.

- The values for $S_0$ and $K_i$ are obtained from the following figures.
$x_1$ is the body station where $\frac{dS_x}{dx}$ first reaches its maximum negative value. $S_x$ is the fuselage cross-sectional area as a function of $x$. 
Wing contribution to $L_v$

• The wing contribution to $L_v$ (non-dimensional) is given by:

$$L_v = \frac{180}{\pi} \left\{ C_L \left[ \left( \frac{C_{l\beta}}{C_L} \right)_{\Lambda_{c/2}} + \left( \frac{C_{l\beta}}{C_L} \right)_A \right] + \Gamma \left( \frac{C_{l\beta}}{\Gamma} \right) K_{M_T} + \theta \tan \Lambda_{c/4} \left( \frac{\Delta C_{l\beta}}{\theta \tan \Lambda_{c/4}} \right) \right\}$$

• $C_L$ is the wing lift coefficient, $\Gamma$ the wing dihedral, $A$ is the aspect ratio, $\theta$ is the wing twist (negative for washout) and $\Lambda_{c/2}$, $\Lambda_{c/4}$ the sweep angle at the half-chord or quarter-chord.

• All the other factors are given by the following figures ($\lambda$ is the taper ratio).
Body contribution to $L_v$

- The body contribution to $L_v$ (non-dimensional) is given by:

\[
L_v = \frac{180\sqrt{A}}{\pi} \frac{d}{b} \left\{ -0.0005 \frac{d}{b} - \frac{2.4\pi}{180} \frac{Z_w}{b} \right\}
\]

- where $Z_w$ is the vertical distance from the wing root quarter-chord to the fuselage centreline, $b$ is the span and $d = \sqrt{\frac{\text{average fuselage cross sectional area}}{0.7854}}$
Wing and body contribution to $N_v$

- The wing contribution to $N_v$ is neglected by the USAF DATCOM.
- The body contribution is given by

\[
N_v = -\frac{180}{\pi} K_N K_R \frac{S_{Bs}}{S} \frac{l_B}{b}
\]

- Where $S$ is the wing area, $S_{Bs}$ is the fuselage side area and $l_B$ is the fuselage length.
- The factors $K_N$ and $K_Rl$ are obtained from the following figures.
$S_{B}$ = Body side area

$w$ = Maximum body width

Reproduced from Reference 1
Wing and body contribution to $L_p$

- The wing and body contribution to $Y_p$ is considered negligible.
- The wing and body contribution to $L_p$ (non-dimensional) is given by:

$$L_p = \left( \frac{\sqrt{1-M^2 C_{lp}}}{\kappa} \right) \kappa \left( C_{lp} \right)_{\Gamma=0}$$

- where $\kappa$ is the ratio of the profile lift curve slope to $2\pi$ and $M$ is the Mach number. Furthermore,

$$\left( \frac{C_{lp}}{\left( C_{lp} \right)_{\Gamma=0}} \right) = 1 - 2 \frac{Z_w}{b/2} \sin \Gamma + 3 \left( \frac{Z_w}{b/2} \right)^2 \sin^2 \Gamma$$

- The factor in brackets is given in the following figures.
\[ \beta = \sqrt{1 - M^2} \]
Wing contribution to $N_p$

- The body contribution to $N_p$ is negligible.
- The wing contribution to $N_p$ (non-dimensional) is given by:

$$N_p = -L_{pw} \tan \alpha - \left( -L_p \tan \alpha - \left( \frac{c_{np}}{C_L} \right)_{C_L=0,M} \right) \left( \frac{\Delta c_{np}}{\theta} \right) \theta$$

- where $\alpha$ is the angle of attack, $L_{pw}$ is the wing contribution to $L_p$. Furthermore, $B = \sqrt{1 - M^2 \cos^2 \Lambda c/4}$

$$\left( \frac{c_{np}}{C_L} \right)_{C_L=0,M} = \frac{A + 4 \cos \Lambda c/4}{AB + 4 \cos \Lambda c/4} \left( AB + \frac{1}{2} \left( AB + \cos \Lambda c/4 \right) \tan^2 \Lambda c/4 \right) \left( \frac{c_{np}}{C_L} \right)_{C_L=0,M=0}$$

$$\left( \frac{c_{np}}{C_L} \right)_{C_L=0,M=0} = -\frac{1}{6} \frac{A + 6 \left( A + \cos \Lambda c/4 \right)}{A + 4 \cos \Lambda c/4} \left[ \left( h_0 - h \right) \frac{\tan \Lambda c/4}{A} + \frac{\tan^2 \Lambda c/4}{12} \right]$$
Calculation of \( \left( \frac{\Delta C_{n_p}}{\theta} \right) \)
Wing contribution to $L_r$

- The wing and body contributions to $Y_r$ are negligible.
- The body contribution to $L_r$ is negligible.
- The wing contribution to $L_r$ is given by

$$L_r = C_L \left( \frac{c_{l_r}}{C_L} \right)_{C_L=0,M} + \left( \frac{\Delta c_{l_r}}{\Gamma} \right) \Gamma + \left( \frac{\Delta c_{l_r}}{\theta} \right) \theta$$

where

$$\left( \frac{\Delta c_{l_r}}{\Gamma} \right) = \frac{1}{12} \frac{\pi A \sin \Lambda_{c/4}}{A + 4 \cos \Lambda_{c/4}}$$

$$\left( \frac{c_{l_r}}{C_L} \right)_{C_L=0,M} = \frac{A \left( 1 - B^2 \right)}{2B \left( AB + 2 \cos \Lambda_{c/4} \right)} + \frac{A B + 2 \cos \Lambda_{c/4}}{A B + 4 \cos \Lambda_{c/4}} \frac{\tan^2 \Lambda_{c/4}}{8} \left( \frac{c_{l_r}}{C_L} \right)_{C_L=0,M=0}$$
\[ \frac{C_{t,I}}{C_L/C_L} = 0 \quad \text{per rad} \]

\[ \frac{\Delta C_{t,I}}{\theta} \quad \text{Rep. from Reference 1} \]

\[ \frac{1}{\text{rad-deg}} \]

\[ \text{NOTE:} \quad \Delta C_{t,I} \text{ per radian} \quad \theta \text{ in degrees} \]

\[ \lambda \geq 0.4 \]

\[ \lambda = 0.2 \]

\[ \lambda = 0 \]

\[ \theta \]

ROOT SECTION ZERO-LIFT LINE

TIP SECTION ZERO-LIFT LINE

ASPECT RATIO, A
Wing contribution to $N_r$

- The body contribution to $N_r$ is negligible.
- The wing contribution to $N_r$ is given by

\[ N_r = \left( \frac{C_{n_r}}{C_L^2} \right) C_L^2 + \left( \frac{C_{n_r}}{C_{D_0}} \right) C_{D_0} \]

- Where the values in brackets are given by the figures overleaf.
\[
\bar{x} = h_0 - h
\]

\( \bar{x} \) is the distance from the c.g. to the a.c., positive for the a.c. aft of the c.g.

\( \bar{c} \) is the mean aerodynamic chord.
DATCOM Rudder derivatives

• The factor $c_2$ for the rudder’s contribution to the fin lift is difficult to calculate.

• The DATCOM proposes the following value for the sideforce rudder derivative:

$$Y_\zeta = c_1 \left( \frac{\alpha \delta_{\text{CL}}}{\alpha \delta_{\text{cl}}} \right) \alpha \delta_{\text{cl}} K' K_b \frac{S_F}{S}$$

• where all the terms were already introduced in lecture 3 and concern the influence of the flaps. They can be calculated using the following figures.
Flap influence terms
DATCOM Rudder derivatives (2)

• DATCOM value for the rolling moment rudder derivative:

\[ L_\zeta = c_1 \left( \frac{\alpha \delta_{CL}}{\alpha \delta_{c_l}} \right) \alpha \delta_{c_l} K' K_b \frac{S_F}{S} \frac{z_F \cos \alpha_e - l_F \sin \alpha_e}{b} \]

• DATCOM value for the rolling moment rudder derivative:

\[ N_\zeta = -c_1 \left( \frac{\alpha \delta_{CL}}{\alpha \delta_{c_l}} \right) \alpha \delta_{c_l} K' K_b \frac{S_F}{S} \frac{z_F \sin \alpha_e + l_F \cos \alpha_e}{b} \]
Propeller normal force

- Propellers at an angle of attack generate both thrust (along the axis) and a normal force (in the propeller’s plane)
- The normal force is given by:
  \[ N_p = \frac{1}{2} \rho V_0^2 s_p C_{N_p} \]
- Where \( s_p \) is the propeller disk area and
  \[ C_{N_p} = e\beta \]
Propeller normal force curve slope

- The propeller normal force curve slope can be determined from:

\[ e = f C_{\psi_0} \]

\[ T_c = \frac{T}{\rho V^2 d^2} \]

where:
- \( e \) is the propeller normal force curve slope
- \( f \) is a function of \( C_{\psi_0} \)
- \( T_c \) is the thrust coefficient
- \( T \) is the thrust
- \( \rho \) is the density
- \( V \) is the velocity
- \( d \) is the propeller diameter
Propeller normal force curve slope (2)

Blade twist angle at 0.75R (in degrees)
Propeller normal force curve slope (3)

\[ SFF = 525 \left( \frac{b}{d} \right)_{0.3} + \left( \frac{b}{d} \right)_{0.6} + 270 \left( \frac{b}{d} \right)_{0.9} \]
Propeller normal force curve slope (4)

• Procedure:
  – Determine $T_c$ and then $f$.
  – Using the blade twist angle determine $\frac{C_{Y_{\psi_0}}}{\psi}$
  – Calculate Side Force Factor (SFF)
  – Determine the correct $C_{Y_{\psi_0}}$ from SFF.
  – More info in
    • Notes on the propeller and slipstream in relation to stability, H. S. Ribner, NACA Wartime Report L-25, 1944
Discussion of propeller normal force

• The propeller normal force generates a yawing moment that is:
  – Destabilising if the propeller lies ahead of the CG (puller propeller)
  – Stabilising if the propeller lies behind the CG (pusher propeller)

• From the $f$ graph, the propeller normal force is a nonlinear function of thrust, since $e = e(T)$.

• However, we can always linearize around a trimmed thrust condition.
Handling the thrust

- We can write the thrust as:
  \[ T = T_e + \tau \]

- where \( T_e \) is the trimmed thrust and \( \tau \) is a small perturbation.

- The propeller normal force curve slope becomes:
  \[ e(T) = e(T_e + \tau) = e(T_e) + \frac{\partial e}{\partial \tau} \tau \]

- Note that we usually ignore propulsion derivatives (i.e. with respect to \( \tau \)) in the lateral equations.
Propeller contributions to \( Y \) and \( N \)

- Contribution to \( Y \):
  \[
  Y_a = -N_p = -\frac{1}{2} \rho V_0^2 s_p e(T) \beta
  \]

- Substituting \( \beta = \beta_e + v/V_0 \) and \( e(T) \):
  \[
  Y_a = Y_{ae} - \frac{1}{2} \rho V_0^2 s_p \beta_e \frac{\partial e}{\partial \tau} \tau - \frac{1}{2} \rho V_0 s_p e(T_e) v
  \]

- Contribution to \( N \):
  \[
  N_a = N_p l_p = \frac{1}{2} \rho V_0^2 s_p l_p e(T) \beta
  \]

- Substituting \( \beta = \beta_e + v/V_0 \) and \( e(T) \):
  \[
  N_a = N_{ae} + \frac{1}{2} \rho V_0^2 s_p l_p \beta_e \frac{\partial e}{\partial \tau} \tau + \frac{1}{2} \rho V_0 s_p l_p e(T_e) v
  \]
Propeller aerodynamic derivatives

- The propeller contributions to the aerodynamic derivatives are:

\[
\begin{align*}
\ddot{Y}_v &= \frac{1}{2} \rho V_0 s_p e(T_e) \\
\ddot{Y}_\tau &= -\frac{1}{2} \rho V_0^2 s_p \beta_e \frac{\partial e}{\partial \tau} \\
\ddot{N}_v &= \frac{1}{2} \rho V_0 s_p l_p e(T_e) \\
\ddot{N}_\tau &= \frac{1}{2} \rho V_0^2 s_p \beta e \frac{\partial e}{\partial \tau}
\end{align*}
\]